

1. (A) 40

2. (D) neither prime nor composite

3. (C)  $\frac{3}{\sqrt{10}}$

4. (B) 5

5. (B)  $\frac{4}{\sqrt{3}}$

6. (A) 960

7. (C) 60 m

8. (C) Two

9. (A)  $30^\circ$

10. (B)  $\frac{2}{3}$

11. (B)  $50^\circ$

12. (D)  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

13. (B)  $\frac{182}{3} \pi$

14. (D)  $1^2, 3^2, 5^2, 7^2, \dots$

15. (C)  $\frac{\pi d^2}{8}$

16. (C) 12

17. (C) 44.2

18. (A)  $\frac{15}{25}$

2

19. (C) A is true, but R is false

20. (D) A is false, but R is true.

SECTION-B

21. ATQ, using distance formula

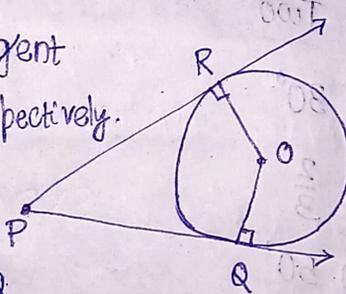
$$(4-1)^2 + (p-0)^2 = 25$$

$$\Rightarrow 9 + p^2 = 25$$

$$\Rightarrow p^2 = 16$$

$$\Rightarrow p = 4 \text{ or } -4$$

22. Given PR and PQ are tangent to the circle at R and Q respectively. OR and OQ are radius.



Since  $OR \perp PR$  and  $OQ \perp PQ$

(As line drawn center to the tangent point is  $\perp$  to the tangent)

$$\therefore \angle ORP = \angle OQP = 90^\circ$$

Since sum of angles of a quadrilateral is  $360^\circ$ .

$$\Rightarrow \angle RPQ + \angle PQO + \angle ORP + \angle ROQ = 360^\circ$$

$$\Rightarrow \angle RPQ + \angle ROQ + 90^\circ + 90^\circ = 360^\circ$$

$$\Rightarrow \angle RPQ + \angle ROQ = 180^\circ$$

Hence PQOR is cyclic.

23.

Given  $\alpha, \beta$  are the zeroes of  $px^2 + qx + r$ ;

So  $\alpha + \beta = -\frac{q}{p}$  and  $\alpha\beta = \frac{r}{p}$

then  $\alpha^3\beta + \beta^3\alpha = \alpha\beta(\alpha^2 + \beta^2)$

$$= \alpha\beta [(\alpha + \beta)^2 - 2\alpha\beta]$$

$$= \frac{r}{p} \left[ \left(-\frac{q}{p}\right)^2 - 2\frac{r}{p} \right]$$

$$= \frac{r}{p} \left[ \frac{q^2 - 2rp}{p^2} \right]$$

$$\Rightarrow \boxed{\alpha^3\beta + \beta^3\alpha = \frac{r}{p^3} (q^2 - 2rp)}$$

24. (a) Given  $\triangle AHK \sim \triangle ABC$

then  $\frac{AK}{AC} = \frac{HK}{BC} = \frac{AH}{AB}$

Given  $AK = 10$  cm,  $BC = 3.5$  cm and  $HK = 7$  cm

Consider  $\frac{AK}{AC} = \frac{HK}{BC}$

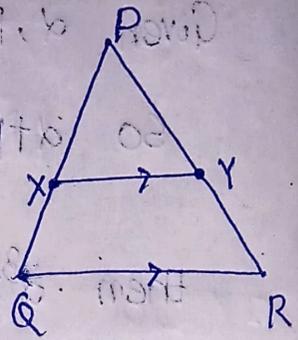
$$\Rightarrow \frac{10}{AC} = \frac{7}{3.5}$$

$$\Rightarrow \boxed{AC = 5 \text{ cm}}$$

24(b.)

Given  $XY \parallel QR$

Then  $\frac{PX}{XQ} = \frac{PY}{YR}$  (By thales theorem) — ①



∴ At Q,  $\frac{PQ}{XQ} = \frac{7}{3}$  and  $PR = 6.3 \text{ cm}$

$$\Rightarrow \frac{PX + XQ}{XQ} = \frac{7}{3}$$

$$\Rightarrow \frac{PX}{XQ} = \frac{7}{3} - 1 = \frac{4}{3} \quad \text{--- ②}$$

From ① & ②

$$\frac{PY}{YR} = \frac{4}{3}$$

$$\Rightarrow \frac{PY}{YR} + 1 = \frac{4}{3} + 1$$

$$\Rightarrow \frac{PR}{YR} = \frac{7}{3}$$

$$\Rightarrow YR = \frac{6.3 \times 3}{7} = 0.9 \times 3$$

$$\Rightarrow \boxed{YR = 2.7}$$

25. (a)

$$\tan A = \frac{4}{3}$$

$$\Rightarrow \frac{p}{b} = \frac{4}{3}$$

$$\Rightarrow \underline{p} = 4K \text{ and } b = 3K$$

$$\text{then } h = \sqrt{p^2 + b^2} = \sqrt{(4K)^2 + (3K)^2} = 5K$$

$$\therefore \sin A = \frac{p}{h} = \frac{4K}{5K} = \frac{4}{5}$$

$$\cos A = \frac{b}{h} = \frac{3K}{5K} = \frac{3}{5}$$

(b) Since  $\cos^2 A + \sin^2 A = 1$   
 $\Rightarrow \cos A = \sqrt{1 - \sin^2 A}$

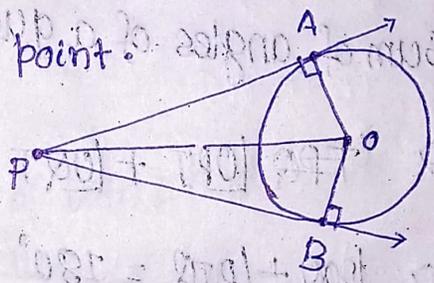
$$\tan A = \frac{\sin A}{\cos A}$$
$$= \frac{\sin A}{\sqrt{1 - \sin^2 A}}$$

**SECTION-C**

26.

Let P be an external point.

PA and PB are two tangents to the circle at A and B respectively.



OA and OB are radius and  $OA \perp PA$

Join OP.

Since  $OA \perp PA$  and  $OB \perp PB$

(as line joining centre to the tangent point is  $\perp$  to the tangent).

In  $\triangle OAP$  and  $\triangle OBP$

OP is common

$OA = OB$

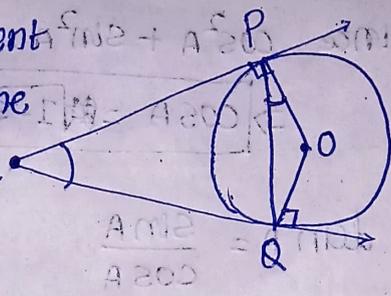
$\angle OAP = \angle OBP$

$\therefore$  By RHS  $\triangle OAP \cong \triangle OBP$ .

Hence  $PA = PB$ .

26(b)

TP and TQ are two tangents from an external point T to the circle at P and Q respectively. Join P and Q.



(6)

Since  $OP \perp TP$  and  $OQ \perp TQ$

(as line segment joining center to the tangent point is  $\perp$  to the tangent)

Sum of angles of a quadrilateral =  $360^\circ$

$$\Rightarrow \angle OPT + \angle OQT + \angle POQ + \angle PTQ = 360^\circ$$

$$\Rightarrow \angle POQ + \angle PTQ = 180^\circ \quad (\text{As } \angle OPT + \angle OQT = 90^\circ + 90^\circ = 180^\circ)$$

~~$\angle PTQ$~~

Again sum of angles is  $180^\circ$

$$\Rightarrow \angle OPQ + \angle OQP + \angle POQ = 180^\circ$$

$$\Rightarrow \angle POQ = 180^\circ - 2\angle OPQ$$

From (i) and (ii)

$$\angle PTQ + 180^\circ - 2\angle OPQ = 180^\circ$$

$$\Rightarrow \boxed{\angle PTQ = 2\angle OPQ}$$

27.

Suppose  $\sqrt{5}$  is an irrational

then  $\sqrt{5} = \frac{p}{q}$ ; where  $\text{HCF}(p, q) = 1$

$$\Rightarrow p^2 = 5q^2 \quad \text{--- (i)}$$

$$\Rightarrow 5 | p^2$$

$$\Rightarrow 5 | p \quad \text{--- (ii)}$$

$\Rightarrow p = 5m$  for some natural number

From (i)

$$(5m)^2 = 5q^2$$

$$\Rightarrow 5m^2 = q^2$$

$$\Rightarrow 5 | q^2$$

$$\Rightarrow 5 | q \quad \text{--- (iii)}$$

From (ii) and (iii)

5 is a common divisor of both  $p$  and  $q$

ie,  $\text{HCF}(p, q) \geq 5$

which is a contradiction.

Hence  $\sqrt{5}$  is a rational.

28.

Radius of a circle ( $r$ ) = 42 cm

Central angle ( $\theta$ ) =  $30^\circ$

Area of the sector

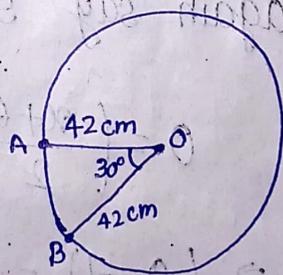
$$\frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 42 \times 42$$

$$= \frac{1}{12} \times 22 \times 6 \times 42$$

$$= 11 \times 42 = 482 \text{ sq. cm.}$$

$$\text{Area of major sector} = \pi r^2 - 482 = \frac{22}{7} \times 42 \times 42 - 482$$



$$= 22 \times 6 \times 42 - 482$$

$$= 132 \times 42 - 482$$

$$= 5544 - 482$$

$$= 5062 \text{ sq. cm.}$$

29.

Given PQRS is a rhombus

Where  $P = (2, 3)$ ,  $Q = (6, 5)$  &  $R = (-2, 1)$

Since in a rhombus all sides length are same.

Since diagonals of rhombus are bisect each other.

So by mid-point formula co-ordinates of

$$O = \left( \frac{-2+2}{2}, \frac{1-3}{2} \right) = (0, -1)$$

again say  $S = (x, y)$ , then by mid-point formula

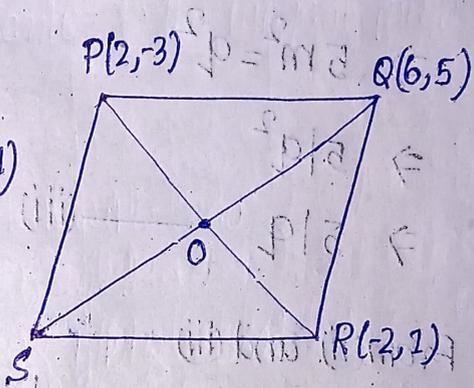
$$O = \left( \frac{x+6}{2}, \frac{y+5}{2} \right)$$

$$\Rightarrow (0, -1) = \left( \frac{x+6}{2}, \frac{y+5}{2} \right)$$

$$\therefore \frac{x+6}{2} = 0 \Rightarrow \boxed{x = -6}$$

$$\frac{y+5}{2} = -1 \Rightarrow \boxed{y = -7}$$

Hence  $S = (-6, -7)$



Two dice are thrown together

Sample space (S) =  $\{(1,1), (1,2), \dots, (6,6)\}$

$$n(S) = 6 \times 6 = 36.$$

(i)  $E_1 =$  Even sum

~~(1,1)~~  
 $= \{(1,2), (2,1), (2,2), (3,3), (3,4), (4,2), (4,3), (4,4), (5,3), (5,4), (5,5), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

	1	2	3	4	5	6
1	<del>(1,1)</del>	<del>(1,2)</del>	<del>(1,3)</del>	<del>(1,4)</del>	<del>(1,5)</del>	<del>(1,6)</del>
2	<del>(2,1)</del>	<del>(2,2)</del>	<del>(2,3)</del>	<del>(2,4)</del>	<del>(2,5)</del>	<del>(2,6)</del>
3	<del>(3,1)</del>	<del>(3,2)</del>	<del>(3,3)</del>	<del>(3,4)</del>	<del>(3,5)</del>	<del>(3,6)</del>
4	<del>(4,1)</del>	<del>(4,2)</del>	<del>(4,3)</del>	<del>(4,4)</del>	<del>(4,5)</del>	<del>(4,6)</del>
5	<del>(5,1)</del>	<del>(5,2)</del>	<del>(5,3)</del>	<del>(5,4)</del>	<del>(5,5)</del>	<del>(5,6)</del>
6	<del>(6,1)</del>	<del>(6,2)</del>	<del>(6,3)</del>	<del>(6,4)</del>	<del>(6,5)</del>	<del>(6,6)</del>

$$n(E_1) = 1 + 3 + 5 + 5 + 3 + 1$$

$$= 18$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{\text{no. of favourable outcomes}}{\text{total no. of outcomes}}$$

$$= \frac{18}{36} = \frac{1}{2}$$

(ii)  $E_2 =$  Even product

~~total~~ sample space - odd product

$= n(S) -$  both are odd in two dice

$$\Rightarrow n(E_2) = n(S) - 3 \times 3 = 36 - 9 = 27$$

$$\Rightarrow n(E_2) = 27$$

$$\therefore P(E_2) = \frac{n(E_2)}{n(S)} = \frac{27}{36} = \frac{3}{4}$$

31-

(a) Consider

$$\frac{\sec^3 \theta}{\sec^2 \theta - 1} + \frac{\operatorname{cosec}^3 \theta}{\operatorname{cosec}^2 \theta - 1}$$

$$= \frac{\sec^3 \theta}{\tan^2 \theta} + \frac{\operatorname{cosec}^3 \theta}{\cot^2 \theta}$$

$$= \frac{\sec^3 \theta}{\left(\frac{\sin^2 \theta}{\cos^2 \theta}\right)} + \frac{\operatorname{cosec}^3 \theta}{\left(\frac{\cos^2 \theta}{\sin^2 \theta}\right)}$$

$$= \sec \theta \cdot \operatorname{cosec}^2 \theta + \operatorname{cosec} \theta \cdot \sec^2 \theta$$

$$= \sec \theta \cdot \operatorname{cosec} \theta (\operatorname{cosec} \theta + \sec \theta)$$

(b)

Given  $\frac{\sec \alpha}{\operatorname{cosec} \beta} = p$  and  $\frac{\tan \alpha}{\operatorname{cosec} \beta} = q$

consider  $(p^2 - q^2) \sec^2 \alpha$

$$= \left( \frac{\sec^2 \alpha}{\operatorname{cosec}^2 \beta} - \frac{\tan^2 \alpha}{\operatorname{cosec}^2 \beta} \right) \sec^2 \alpha$$

$$= \frac{\sec^2 \alpha - \tan^2 \alpha}{\operatorname{cosec}^2 \beta} \times \sec^2 \alpha$$

$$= \left( \frac{\sec \alpha}{\operatorname{cosec} \beta} \right)^2$$

$$= p^2$$

Hence  $(p^2 - q^2) \sec^2 \alpha = p^2$

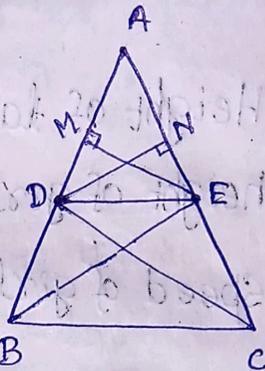
(11)

Section-D

$$\frac{AE}{EC} = \frac{AD}{DB}$$

32. (a) Basic proportionality theorem

Let  $DE \parallel BC$  where D and E are two distinct points on AB and AC respectively.



Join DC and BE

Draw  $EM \perp AD$  and  $DN \perp AE$

$$\text{ar}(\triangle ADE) = \frac{1}{2} \times AD \times EM \quad \text{--- (i)}$$

$$\text{ar}(\triangle AED) = \frac{1}{2} \times AE \times DN \quad \text{--- (ii)}$$

Since  $\triangle BDE$  and  $\triangle CED$  are lies in between pair of parallel lines  $DE$  and  $BC$  and sharing common base  $DE$  are same area

$$\text{i.e., } \text{ar}(\triangle BDE) = \text{ar}(\triangle CED)$$

$$\text{Again } \text{ar}(\triangle BDE) = \frac{1}{2} \times DB \times EM \quad \text{--- (iii)}$$

$$\text{ar}(\triangle CED) = \frac{1}{2} \times EC \times DN \quad \text{--- (iv)}$$

Consider

$$\text{ar}(\triangle BDE) = \text{ar}(\triangle CED)$$

$$\Rightarrow \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle CED)}$$

$$\Rightarrow \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times DB \times EM} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN}$$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

SECTION-D

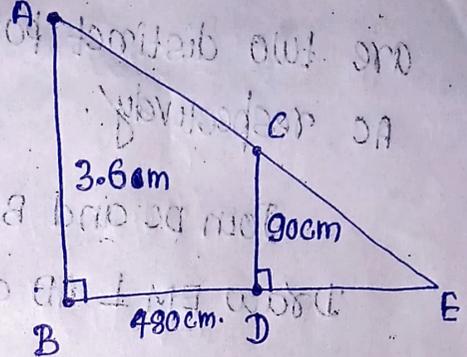
(b).

Height of lamp = 3.6 m = 360 cm

height of girl = 90 cm

Speed of girl = 1.2 m/s

= 120 cm/s



In 1's distance travelled

$$= 120 \times 1 = 120 \text{ cm}$$

In 4's distance travelled = 480 cm

Shadow of girl = DE

AS  $CD \parallel AB$ , by ABPT

$$\frac{EB}{ED} = \frac{AB}{CD}$$

$$\Rightarrow \frac{EB}{ED} = \frac{360}{90} = 4$$

$$\Rightarrow \frac{EB}{ED} - 1 = 3$$

$$\Rightarrow \frac{BD}{ED} = 3$$

$$\Rightarrow ED = \frac{480}{3}$$

$$\Rightarrow \boxed{ED = 160 \text{ cm}}$$

Hence shadow of girl is 1.6 m.

33.

13

Given data of SBI policy holder:

Age (in yrs) $I$	Number of policy holders ( $f$ )	Cumulative frequency
15-20	2	2
20-25	4	6
25-30	18	24
30-35	21	45
35-40	33	78
40-45	11	89
45-50	3	92
50-55	6	98
55-60	2	100

Here highest frequency ( $f_1$ ) = 33

modal class: 35-40

 $l$  = lower limit of the modal class = 35 $f_0$  = preceding frequency of the modal class  
= 21 $f_2$  = succeeding frequency of the modal class = 11 $h$  = Interval length = 5

$$\therefore \text{Grouped frequency mode} = l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 35 + \frac{33 - 21}{2(33) - 21 - 11} \times 5$$

= 35 + 12/33 x 5

= 35 + 60/33

= 35 + 20/11

≈ 35 + 1.82

⇒ mode = 36.82

For median class: (100/2)<sup>th</sup> term = 35-40

Total frequency (N) = 100

l = lower limit of the median class = 35

C<sub>f</sub> = Cumulative frequency of the preceding median class = 45

f = frequency of the median class = 33

h = Interval length = 5

∴ median class = l + (N/2 - C<sub>f</sub>) / f x h

= 35 + (100/2 - 45) / 33 x 5

= 35 + 25/33

≈ 35 + 0.76

⇒ median = 35.76

34.

Given a pair of linear equations

$L_1: x - 5y = 6$

$L_2: 2x - 10y = 12$

x	1	6
y	-1	0

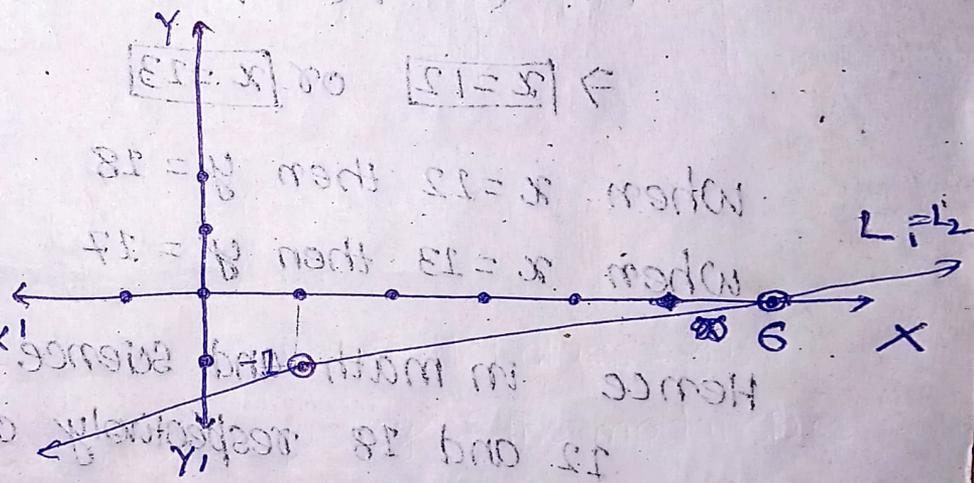
$L_2: 2x - 10y = 12$

$\Rightarrow x - 5y = 6$

which represent same equation

Since  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

Hence both lines are coincidence line



35.

(a) Let Anamika marks in math's =  $x$

in Science =  $y$

ATQ,  $x + y = 30$  (i)

again  $(x+2)(y-3) = 210$  (ii)

$\Rightarrow xy - 3x + 2y - 6 = 210$

$\Rightarrow 3x + 2y = 6$

(15)

From (i)  $y = 30 - x$ Replace in eq<sup>n</sup> (ii)

$$(x+2)(30-x-3) = 210$$

$$\Rightarrow (x+2)(27-x) = 210$$

$$\Rightarrow -x^2 + 25x + 54 = 210$$

$$\Rightarrow x^2 - 25x + 156 = 0$$

$$\Rightarrow x^2 - 12x - 13x + 156 = 0$$

$$\Rightarrow (x-12)(x-13) = 0$$

$$\Rightarrow \boxed{x=12} \text{ or } \boxed{x=13}$$

When  $x=12$  then  $y=18$ When  $x=13$  then  $y=17$ 

Hence in math and science either she got 12 and 18 respectively or 13 and 17 respectively.

(b). In a right angle triangle

Let the dimension of shortest side =  $x$  cmBy ATQ its hypotenuse =  $2x + 6$  cmand other side =  $3x - 6$  cm

By pythagorean theorem

$$h^2 = p^2 + b^2$$

$$\Rightarrow (2x+6)^2 = x^2 + (3x-6)^2$$

(16)

⇒ 4x² + 36 + 24x = x² + 9x² + 36 - 36x

⇒ 6x² - 60x = 0

⇒ x(x - 10) = 0

⇒ x = 10 (as x ≠ 0)

∴ Shortest side = 10 cm

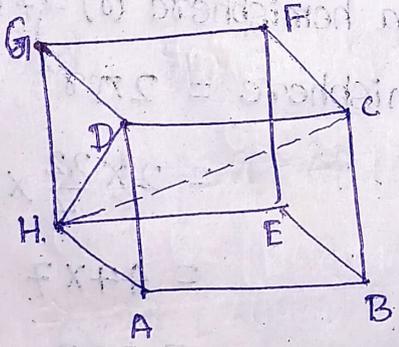
hypotenuse = 26 cm

and other side = 24 cm

SECTION-E

36.

Given a Rubik's cube



(i) Edge of Rubik's cube = 6 cm

Here HD is a side diagonal and HC is a main diagonal.

In ~~right~~ since ΔHAD and ΔHDC are two right angle triangle.

Since  $HD^2 = HA^2 + AD^2$   
 $= 6^2 + 6^2 = 2 \times 6^2$

⇒ HD = 6√2 cm

Again in  $\triangle HDC$  RAT,

$$HC^2 = HD^2 + DC^2$$

$$= 2 \times 6^2 + 6^2$$

$$= 3 \times 6^2$$

$$\Rightarrow HC = 6\sqrt{3} \text{ cm}$$

(ii) If the length of the edge is 7 cm.

Volume of Rubik's cube = (Side)<sup>3</sup> cu.cm

$$= 7^3 \text{ cu.cm}$$

$$= 147 \text{ cu.cm}$$

(iii) (a) Base radius of a hemisphere (r) = 7 cm

$$\text{CSA of hemisphere} = 2\pi r^2$$

$$= 2 \times \frac{22}{7} \times 7^2$$

$$= 44 \times 7$$

$$= 308 \text{ sq. cm.}$$

OR

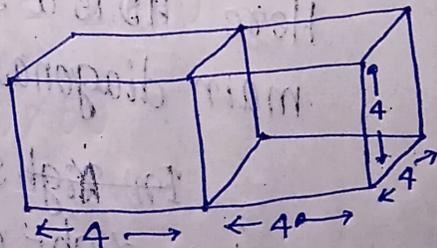
(b) Two cubes of edge 4 cm

are joined end-to-end.

Surface area

$$= 2(lb + bh + hl)$$

$$= 2[8 \times 4 + 4 \times 4 + 4 \times 8] = 160 \text{ sq. cm.}$$



37.

Total loan = ₹ 1,18,000

First instalment (a) = ₹ 1000

Increase every month (d) = ₹ 100

(i) Amount paid in n<sup>th</sup> instalment (a<sub>n</sub>) = a + (n-1)d

So amount paid in 30<sup>th</sup> instalment (a<sub>30</sub>)

= a + (30-1)d

= 1000 + 29 × 100

= ₹ 3900

(ii) If total number of instalments is 40.

ATQ, S<sub>40</sub> = 1,18,000

~~⇒  $\frac{40}{2} [2a + (40-1)d] = 118000$~~

~~⇒  $20 [2(1000) + 39(100)] = 118000$~~

~~⇒ 1180~~

⇒  $\frac{40}{2} [a + a_{40}] = 1,18,000$

⇒ a<sub>40</sub> = 5900 - 1000

⇒ a<sub>40</sub> = 4900

Hence amount paid in the last instalment in ₹ 4900.

(ii) (a) Total amount upto 30<sup>th</sup> instalment

$$= S_{30} = \frac{30}{2} [2a + (n-1)d]$$

$$= 15 [2(1000) + 29(100)]$$

Amount paid in 30 instalment (a)

$$= 15 [2000 + 2900]$$

30 instalment (a)

$$= ₹ 51,450$$

(ii) (b) 10<sup>th</sup> instalment (a<sub>10</sub>) = a + (n-1)d

$$= 1000 + 9(100)$$

10<sup>th</sup> instalment = ₹ 1900 (ii)

To find last instalment

$$S_n = 1,18,000$$

$$\Rightarrow \frac{n}{2} [2a + (n-1)d] = 1,18,000$$

$$\Rightarrow n [2(1000) + (n-1)100] = 2,36,000$$

$$\Rightarrow n [20 + n - 1] = 2,360$$

$$\Rightarrow n(n+19) = 40 \times 59$$

$$\Rightarrow \boxed{n=40}$$

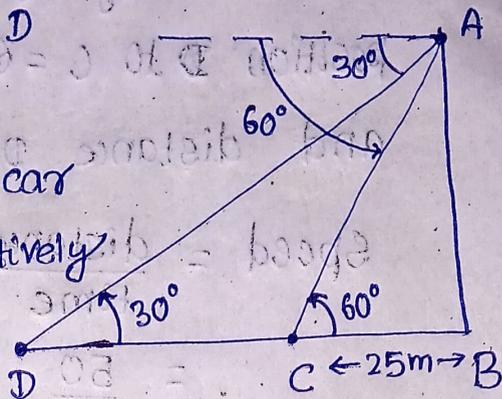
$$\therefore a_{40} = 1000 + 39 \times 100 = 4900$$

$$\therefore \frac{a_{10}}{a_{40}} = \frac{1900}{4900} = \frac{19}{49}$$

38.

Position of car from D  
to C takes 6 seconds.

Angle of depression of car  
at D and C are respectively  
 $30^\circ$  and  $60^\circ$ .



i.e.  $\angle ACB = 60^\circ$  and  $\angle ADC = 30^\circ$

(i)  $BC =$  distance of car from foot of the  $\perp$  to the  
position C = 25 m.

In  $\triangle ABC$ , RAT

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\Rightarrow AB = \sqrt{3} (25) = 25\sqrt{3} \text{ m.}$$

$\therefore$  height of the building is  $25\sqrt{3}$  m.

(ii) In  $\triangle ABD$ , RAT

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\Rightarrow BC + CD = \frac{AB}{\tan 30^\circ} = \frac{25\sqrt{3}}{\frac{1}{\sqrt{3}}} = 75 \text{ m}$$

$$\Rightarrow CD = 75 \text{ m} - 25 \text{ m}$$

$$\Rightarrow \boxed{CD = 50 \text{ m}}$$

Distance between two position of the car = 50 m.

(iii) (a)

Time taken by the car to reach from

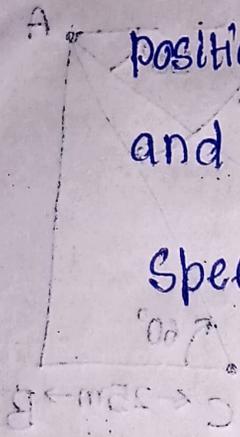
position D to C = 6 seconds.

and distance DC = 50 m

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{50}{6}$$

$$\approx 8.33 \text{ m/s}$$



So distance DB = 50 + 25 = 75 m

$$\text{time taken to cover} = \frac{DB}{\text{Speed}}$$

$$= \frac{75}{\left(\frac{50}{6}\right)} = \frac{75 \times 6}{50}$$

$$= \frac{450}{50} = 9 \text{ seconds.}$$

(b) Distance of the observer from the car

at  $60^\circ$ , is  $\cos 60^\circ = \frac{CB}{AC}$

$$\Rightarrow AC = \frac{25}{\left(\frac{1}{2}\right)}$$

$$\Rightarrow \boxed{AC = 50 \text{ m}}$$