CBSE BOARD - 2026 (REG)

MATHEMATICS (041) : SET - 2

SECTION - A

1. (B) $y = \cot^{-1} x$

2. (B) $\begin{bmatrix} 1/3 & 0 \\ 3 & 4/3 \end{bmatrix}$

3. (C) 4

4. (C) $e^{x} e^{e^{x}}$

5. (B) $-\frac{\pi}{4}$

6. (B) 6

7. (C) $\frac{1}{4} \sin^{-1}\left(\frac{4x}{5}\right) + c$

8. (D) $-\frac{\pi}{4}$

9. (D) 2

10. (A) 2 sq. units

11. (B) At only two points

12. (D) 4

13. (B) $p = \frac{-8}{3}$; $q = \frac{20}{3}$

14. (C) $3p = q$

15. (A) $\frac{\sqrt{53}}{2}$ sq. units

16. (A) unbounded in 1^{st} quadrant

17. (C) $x(1+y^2)\,dx - y(1+x^2)\,dy = 0$

18. (B) $\frac{244}{250}$

19. (B)

20. (A)

SECTION-B

21. Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix} = \hat{i}(2-3) - \hat{j}(-8+6) + \hat{k}(4-2)$$

$$= -\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(-1)^2 + 2^2 + 2^2} = 3$$

$$\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{-\hat{i} + 2\hat{j} + 2\hat{k}}{3}$$

The vector $\perp$r to $\vec{a}$ and $\vec{b}$ $= \lambda \hat{n}$

$\Rightarrow |\lambda \hat{n}| = 5$ (given)

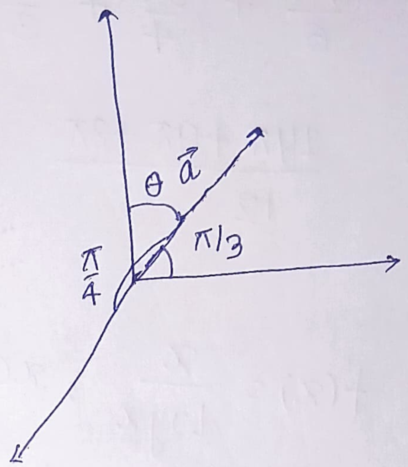
$\Rightarrow \boxed{\lambda = \pm 5}$

Hence required vector $= \lambda \hat{n} = \pm\left(-\frac{5}{3}\hat{i} + \frac{10}{3}\hat{j} + \frac{10}{3}\hat{k}\right)$

21 (b) Given $\vec{a}$ making angle $\frac{\pi}{4}$ with x-axis,

$\frac{\pi}{3}$ with y-axis

and θ with z-axis.



Here $l = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$m = \cos\frac{\pi}{3} = \frac{1}{2}$

$n = \cos\theta$, θ is acute

$\therefore\ l^2 + m^2 + n^2 = 1$

$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = \frac{1}{4}$

$\Rightarrow \cos\theta = \pm\frac{1}{2}$

$\Rightarrow \cos\theta = \frac{1}{2}$; as θ is acute

$\Rightarrow \boxed{\theta = \frac{\pi}{3}}$

$\therefore\ \boxed{n = \frac{1}{2}}$

Equation of line passing through origin $(0,0,0)$ and having d.c.s. l, m and n is given by:

$$\frac{x-0}{l} = \frac{y-0}{m} = \frac{z-0}{n}$$

$$\Rightarrow \frac{x}{1/\sqrt{2}} = \frac{y}{1/2} = \frac{z}{1/2} \Rightarrow \boxed{\sqrt{2}x = 2y = 2z}$$

$\boxed{22.}$ $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right) + \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$ (4)

$$= \frac{5\pi}{6} + \frac{\pi}{3} + \tan^{-1}(-1) + \tan^{-1}(-\sqrt{3})$$

$$= \frac{7\pi}{6} + \frac{3\pi}{4} + \frac{2\pi}{3}$$

$$= \frac{14\pi + 9\pi + 8\pi}{12} = \frac{31\pi}{12}$$

$\boxed{23.}$ $f(x) = \frac{x}{\log x}$; $x \in (0,1) \cup (1,\infty)$

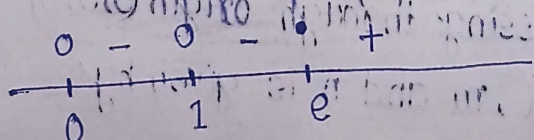
$$f'(x) = \frac{1 \cdot \log x - x \cdot \frac{1}{x}}{\log^2 x} = \frac{(\log x - 1)}{\log^2 x}$$

For increasing: $f'(x) \geq 0$

$$\Rightarrow \frac{\log x - 1}{\log^2 x} \geq 0$$

$\log x = 0 \Rightarrow \boxed{x = 1}$

$\log x - 1 = 0 \Rightarrow \boxed{x = e}$

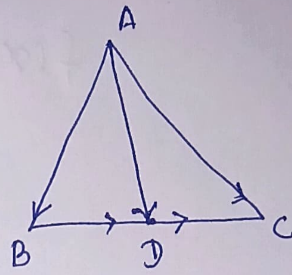


By wavy curve method $x \geq e$

Hence required interval $[e, \infty)$.

24. Given $\vec{AB} = 2\hat{i}+3\hat{j}+\hat{k}$

and $\vec{AC} = 2\hat{i}+\hat{j}-\hat{k}$ of a ΔABC.



Then by triang law of vector addition

$\vec{AB}+\vec{BC} = \vec{AC}$

$\Rightarrow \vec{BC} = \vec{AC} - \vec{AB}$

$= (2\hat{i}+\hat{j}-\hat{k})-(2\hat{i}+3\hat{j}+\hat{k})$

$= -2\hat{j}-2\hat{k}$

Say 'D' be the mid-point of BC and ~~so~~ $\vec{AD}$ is the median.

again by triangle law of vector addition

$\vec{AB}+\vec{BD} = \vec{AD}$

$\Rightarrow \vec{AD} = \vec{AB}+\frac{1}{2}\vec{BC}$

$= (2\hat{i}+3\hat{j}+\hat{k})+\frac{1}{2}(-2\hat{j}-2\hat{k})$

$= 2\hat{i}+3\hat{j}+\hat{k}-\hat{j}-\hat{k}$

$\Rightarrow \boxed{\vec{AD} = 2\hat{i}+2\hat{j}}$

OR

By triangle law of vector addition

Since $\vec{AD} = \vec{AB}+\frac{1}{2}\vec{BC}$

$= \vec{AB}+\frac{1}{2}(\vec{AC}-\vec{AB})$

$\Rightarrow \boxed{\vec{AD} = \frac{1}{2}(\vec{AB}+\vec{AC})}$

So, $\vec{AD} = \frac{1}{2}\left((2\hat{i}+3\hat{j}+\hat{k})+(2\hat{i}+\hat{j}-\hat{k})\right) = 2\hat{i}+2\hat{j}$

25. (a) Given

$$f(x) = \begin{cases} \dfrac{\cos x}{-x+\frac{\pi}{2}} ; & x \neq \frac{\pi}{2} \\ 1 ; & x = \frac{\pi}{2} \end{cases}$$

LHL: $f(\frac{\pi}{2}^-) = \lim_{h\to 0} f(\frac{\pi}{2}-h); h>0$

$$= \lim_{h\to 0} \frac{\cos(\frac{\pi}{2}-h)}{-(\frac{\pi}{2}-h)+\frac{\pi}{2}}$$

$$= \lim_{h\to 0} \frac{\sin h}{h} = 1$$

RHL: $f(\frac{\pi}{2}^+) = \lim_{h\to 0} f(\frac{\pi}{2}+h); h>0$

$$= \lim_{h\to 0} \frac{\cos(\frac{\pi}{2}+h)}{-(\frac{\pi}{2}+h)+\frac{\pi}{2}}$$

$$= \lim_{h\to 0} \frac{-\sin h}{-h} = 1$$

and $f(\frac{\pi}{2}) = 1$ (given)

Since $f(\frac{\pi}{2}^-) = f(\frac{\pi}{2}^+) = f(\frac{\pi}{2})$

Hence $f(x)$ is continuous at $x = \frac{\pi}{2}$

25 (b) Given

$$f(x) = \begin{cases} x-1, & x < 2 \\ 2x-3, & x \geq 2 \end{cases}$$

LHD: $f'(2^-) = \lim_{h \to 0} \frac{f(2-h) - f(2)}{-h}$; $h > 0$

$= \lim_{h \to 0} \frac{((2-h)-1) - (2(2)-3)}{-h}$

$= \lim_{h \to 0} \frac{(1-h)-1}{-h}$

$= \lim_{h \to 0} \frac{-h}{-h}$

$= \lim_{h \to 0} 1 = 1$

RHD: $f'(2^+) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$, $h > 0$

$= \lim_{h \to 0} \frac{(2(2+h)-3) - (2(2)-3)}{h}$

$= \lim_{h \to 0} \frac{(2h+1) - 1}{h}$

$= \lim_{h \to 0} \frac{2h}{h}$

$= \lim_{h \to 0} 2 = 2$

Since $f'(2^-) \neq f'(2^+)$

Hence $f(x)$ is not differentiable at $x=2$.

SECTION-C

26. Denote $P(D)$ = Probability of holding a debate competition $= \frac{1}{3}$

$P(Q) =$ " " Quiz Competition $= \frac{2}{3}$

Team A has 4 girls and 6 boys and

Team B has 7 girls and 3 boys.

E_1 = Selecting 1 girl and 1 boy from team A

$$\therefore P(E_1) = \frac{{}^4C_1 \times {}^6C_1}{{}^{10}C_2} = \frac{4 \times 6}{\frac{10 \times 9}{2 \times 1}} = \frac{8}{15}$$

E_2 = Selecting one girl and one boy from team B

$$\therefore P(E_2) = \frac{{}^7C_1 \times {}^3C_1}{{}^{10}C_2} = \frac{7 \times 3}{\frac{10 \times 9}{2 \times 1}} = \frac{7}{15}$$

Hence probability of selecting 2 students of which 1 girl and 1 boy $= \frac{1}{3} \times \frac{8}{15} + \frac{2}{3} \times \frac{7}{15}$

$$= \frac{22}{45}$$

27. Given $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$$\therefore A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

$\therefore\ A^2-4A-5I$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9-4-5 & 8-8-0 & 8-8-0 \\ 8-8-0 & 9-4-5 & 8-8-0 \\ 8-8-0 & 8-8-0 & 9-4-5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Method-II

Characteristic Eq^n of A is $|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 2 & 2 \\ 2 & 1-\lambda & 2 \\ 2 & 2 & 1-\lambda \end{vmatrix} = 0$$

Expansion through 1st row,

$$\Rightarrow (1-\lambda)\begin{vmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ 2 & 1-\lambda \end{vmatrix} + 2\begin{vmatrix} 2 & 1-\lambda \\ 2 & 2 \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\left((1-\lambda)^2-4\right) - 2(2-2\lambda-4) + 2(4-2+2\lambda) = 0$$

$$\Rightarrow (1-\lambda)^3 - 4 + 4\lambda + 4 + 4\lambda + 4 + 4\lambda = 0$$

$$\Rightarrow 1-3\lambda+3\lambda^2-\lambda^3+12\lambda+4=0$$

$\Rightarrow \lambda^3 - 3\lambda^2 - 9\lambda - 5 = 0$

$\Rightarrow (\lambda+1)(\lambda^2 - 4\lambda - 5) = 0$

~~Since~~ So, A satisfies its own characteristic Eqn.

So $\quad (A+I)(A^2 - 4A - 5I) = 0$

$\Rightarrow \boxed{A^2 - 4A - 5I = 0} \quad (\text{AS } A \neq I)$

28. (a) Given $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$

$f : A \to B$ by

$f(x) = \dfrac{x-2}{x-3}$

<u>For one-one</u>

Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in A$

$\Rightarrow \dfrac{x_1 - 2}{x_1 - 3} = \dfrac{x_2 - 2}{x_2 - 3}$

$\Rightarrow \dfrac{x_1 - 3 + 1}{x_1 - 3} = \dfrac{x_2 - 3 + 1}{x_2 - 3}$

$\Rightarrow 1 + \dfrac{1}{x_1 - 3} = 1 + \dfrac{1}{x_2 - 3}$

$\Rightarrow x_1 - 3 = x_2 - 3$

$\Rightarrow \boxed{x_1 = x_2}$

$\therefore$ f is one-one

For onto

Let $y = f(x)$ ~~for $x \in A$~~

$\Rightarrow y = \frac{x-2}{x-3}$

$\Rightarrow y = 1 + \frac{1}{x-3}$

$\Rightarrow x-3 = \frac{1}{y-1}$.

$\Rightarrow \boxed{x = 3 + \frac{1}{y-1}}$

$\Rightarrow \boxed{x = \frac{3y-2}{y-1}}$

When $y \in B = \mathbb{R} - \{1\}$

$y - 1 \in \mathbb{R} - \{0\}$

$\Rightarrow \frac{1}{y-1} \in \mathbb{R} - \{0\}$

$\Rightarrow 3 + \frac{1}{y-1} \in \mathbb{R} - \{3\}$

$\Rightarrow x \in \mathbb{R} - \{3\}$

$\therefore$ For each $y = f(x) \in \mathbb{R} - \{1\}$ there exist $x \in \mathbb{R} - \{3\}$ such that $y = f(x)$.

$\therefore$ f is onto.

Hence f is a bijective function.

28. (b) Given n be a fixed positive integer.

$R = \{(x,y) : x-y$ is divisible by n, $x,y \in \mathbb{Z}\}$.

For reflexive

Since $\forall x \in \mathbb{Z}$

$x - x = 0$, is divisible by n

$\Rightarrow (x,x) \in R$; $\forall x \in \mathbb{Z}$

$\therefore$ R is a reflexive

For symmetric

Let $(x,y) \in R$

$\Rightarrow x-y$ is divisible by n

$\Rightarrow (-1)(y-x)$ is divisible by n

$\Rightarrow y-x$ is divisible by n as HCF$(n,-1)=1$

$\Rightarrow (y,x) \in R$

$\therefore$ R is symmetric

For transitive

Let (x,y) and $(y,z) \in R$

$\Rightarrow x-y$ and $y-z$ are divisible by n

$\Rightarrow x-y = nk_1$ and $y-z = nk_2$ for some $k_1, k_2 \in \mathbb{Z}$

$\Rightarrow x-z = (x-y)+(y-z) = nk_1 + nk_2$

$\Rightarrow x-z = n(k_1+k_2)$; $k_1+k_2 \in \mathbb{Z}$.

$\Rightarrow x-z$ is also divisible by n

$\therefore$ R is transitive on $\mathbb{Z}$

Hence R is an equivalence relation on $\mathbb{Z}$.

29. Given L.P.P.

Minimize $(Z) = 20x + 10y$

S.t.C.

$x + 2y \leq 40$

$3x + y \geq 30$

$4x + 3y \geq 60$

$x, y \geq 0$

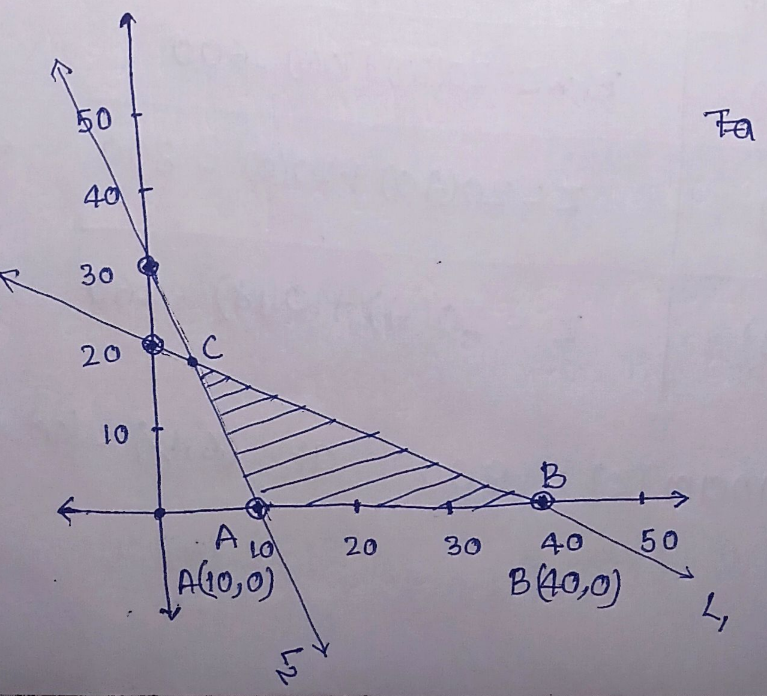
Consider L_1: $x + 2y = 40$

x	0	40
y	20	0

and L_2: $3x + y = 30$

x	0	10
y	30	0

Draw the above straight line L_1 and L_2 in xy-plane



Take the common region of the given constraint and non-negative restriction. The above shaded region ABC is the required feasible region.

Here cornor points are $A=(30,0)$, $B=(40,0)$ and c

<u>To Find c</u>

c is the intersecting point of L_1 and L_2

$L_1 \times 1$: $x+2y=40$

$L_2 \times 2$: $6x+2y=60$

(-) (-) (-)

$-5x=-20$

$\Rightarrow \boxed{x=4}$

From L_2 ; $y=30-3x$

$=30-3(4)$

$\Rightarrow \boxed{y=18}$

$\therefore$ $c=(4,18)$.

Coornor point	$Z=20x+10y$
$A=(30,0)$	$Z=20(30)+10(0)=600$
$B=(40,0)$	$Z=20(40)+10(0)=800$ ←
$c=(4,18)$	$Z=20(4)+10(18)=260$

$\therefore$ maximum $(Z)=800$ at $(x,y)=(40,0)$.

30. (a) Given $xy = e^{x-y}$

Differentiate w.r.t. x both side

$$\Rightarrow \frac{d}{dx}(xy) = \frac{d}{dx} e^{x-y}$$

$$\Rightarrow 1 \cdot y + x \cdot \frac{dy}{dx} = \left(1 - \frac{dy}{dx}\right) e^{x-y}$$

$$\Rightarrow \left(x + e^{x-y}\right) \frac{dy}{dx} = e^{x-y} - y$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{e^{x-y} - y}{e^{x-y} + x}}$$

(b) Let $x = \cos 2\theta$, where $-1 < x < 1$

then
$$\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} = \frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}$$

$$= \frac{\sqrt{2\cos^2\theta} - \sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta} + \sqrt{2\sin^2\theta}}$$

$$= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$$= \frac{1 - \tan\theta}{1 + \tan\theta}$$

$$= \tan\left(\frac{\pi}{4} - \theta\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right) = \cancel{\tan} \frac{\pi}{4} - \theta = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}(x)$$

$$\Rightarrow \frac{d}{dx}\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right) = \frac{d}{dx}\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x\right) = -\frac{1}{2}\frac{-1}{\sqrt{1-x^2}}$$

$$= \frac{1}{2\sqrt{1-x^2}}$$

and $\frac{d}{dx}\cos^{-1}(x^2) = \frac{-1}{\sqrt{1-(x^2)^2}}2x = \frac{-2x}{\sqrt{1-x^4}}$

Hence $\frac{d}{d\cos^{-1}(x^2)}\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right) = \dfrac{\frac{d}{dx}\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)}{\frac{d}{dx}\cos^{-1}(x^2)}$

$$= \frac{\frac{1}{2\sqrt{1-x^2}}}{\frac{-2x}{\sqrt{1-x^4}}}$$

$$= \frac{\sqrt{1-x^4}}{x\sqrt{1-x^2}}$$

$\boxed{31.}$ (a) Given straight line

$$\frac{x-2}{3} = \frac{1-y}{2} = \frac{z-3}{2} = \lambda \text{ (say)}$$

$$\Rightarrow x = 3\lambda+2 \;;\; y = 1-2\lambda \;;\; z = 2\lambda+3$$

Distance between $(3\lambda+2, 1-2\lambda, 2\lambda+3)$ and $(1,2,3)$ is $\sqrt{2}$.

$$\Rightarrow (3\lambda+2-1)^2 + (1-2\lambda-2)^2 + (2\lambda+3-3)^2 = 2$$

$\Rightarrow 9\lambda^2+1+6\lambda + 4\lambda^2+1+8\lambda + 4\lambda^2 = 2$

$\Rightarrow 17\lambda^2 + 14\lambda = 0$

$\Rightarrow \lambda(17\lambda + 14) = 0$

$\Rightarrow \boxed{\lambda = 0}$ or $\boxed{\lambda = -\frac{14}{17}}$

Cas-I : When $\lambda = 0$, point becomes $(2, 1, 3)$

CaseII : When $\lambda = -\frac{14}{17}$, point becomes $\left(\frac{4}{17}, \frac{37}{17}, \frac{31}{17}\right)$

(b) Given lines

$L_1: \vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$

and $L_2: \vec{r} = (\hat{i} + 2\hat{k} - \hat{j}) + \mu(2\hat{i} - 5\hat{k} + 4\hat{j})$

i.e., $x\hat{i} + y\hat{j} + z\hat{k} = (4+\lambda)\hat{i} + (-1+2\lambda)\hat{j} - 3\lambda\hat{k}$

$\Rightarrow \boxed{\frac{x-4}{1} = \frac{y-(-1)}{2} = \frac{z}{-3}}$

and $x\hat{i} + y\hat{j} + z\hat{k} = (1+2\mu)\hat{i} + (2-5\mu)\hat{k} + (-1+4\mu)\hat{j}$

$\Rightarrow \boxed{\frac{x-1}{2} = \frac{z-2}{-5} = \frac{y+1}{4}}$

Here $x_1 = 4,\ y_1 = -1,\ z_1 = 0\ ;\ a_1 = 1,\ b_1 = 2,\ c_1 = -3$

$x_2 = 1,\ y_2 = -1,\ z_2 = 2\ ;\ a_2 = 2,\ b_2 = -5,\ c_2 = 4$

$\therefore$ distance between L_1 and L_2 is

$$d = \frac{\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(a_1b_2-a_2b_1)^2+(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2}}$$

$$= \frac{\begin{vmatrix} 4-1 & -1-(-1) & 0-2 \\ 1 & 2 & -3 \\ 2 & -5 & 4 \end{vmatrix}}{\sqrt{(1(-5)-2(2))^2+(2(4)-(-5)(-3))^2+((-3)2-4(1))^2}}$$

$$= \frac{\{3(8-15)-0-2(-5-4)\}}{\sqrt{81+49+100}} = \frac{3}{\sqrt{230}}$$

SECTION-D

$\boxed{32.}$ (a) Given D.E as

$$ye^y\,dx = (y^3+2xe^y)\,dy \; ; \; y(0)=1$$

$$\Rightarrow \frac{dx}{dy} = \frac{y^2}{e^y} + \frac{2}{y}x$$

$$\Rightarrow \frac{dx}{dy} + \left(-\frac{2}{y}\right)x = \frac{y^2}{e^y} \quad \text{——— (i)}$$

Which is a Linear D.E.

$$I.F = e^{\int \left(\frac{-2}{y}\right) dy} = e^{-2\ln y} = \frac{1}{y^2}.$$

Then solution of the given DE is

$$I.F \times x = \int I.F \times \frac{y^2}{e^y}\, dy + c$$

$$\Rightarrow \frac{1}{y^2}\, x = \int \frac{1}{y^2} \frac{y^2}{e^y}\, dy + c$$

$$= -e^{-y} + c$$

$$\Rightarrow \boxed{x = y^2 (c - e^{-y})}$$

Apply initial condition at $x = 0 ; y = 1$, we get

$$\therefore 0 = c - e^{-1} \Rightarrow \boxed{c = \frac{1}{e}}$$

$$\therefore \boxed{x = y^2 (e^{-1} - e^{-y})}$$

(b) Given D.E $(x^3 - 3xy^2)\, dx = (y^3 - 3x^2 y)\, dy$

$$\Rightarrow \frac{dy}{dx} = \frac{x(x^2 - 3y^2)}{y(y^2 - 3x^2)} \quad \text{———} (1)$$

Here $f(x, y) = \dfrac{x^3 - 3xy^2}{y^3 - 3x^2 y}$

$$\Rightarrow f(tx, ty) = f(x, y)$$

$\therefore f(x, y)$ is a homogeneous function of degree 0.

$\therefore$ Equation (i) is a homogeneous D.E.

Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x^3 - 3x v^2 x^2}{v^3 x^3 - 3x^2 v x}$$

$$= \frac{1 - 3v^2}{v^3 - 3v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1-3v^2}{v^3-3v} - v$$

$$= \frac{1 - 3v^2 - v^4 + 3v^2}{v^3 - 3v}$$

$$= \frac{1 - v^4}{v^3 - 3v}$$

By variable separable

$$\frac{v^3 - 3v}{1 - v^4} dv = \frac{1}{x} dx$$

Integrate both side ~~w.r.t x~~.

$$\int \frac{v^3 - 3v}{(1+v^2)(1-v^2)} dv = \int \frac{1}{x} dx \quad \text{——— (ii)}$$

By partial fraction

$$\frac{v^3 - 3v}{(1+v^2)(1-v)(1+v)} = \frac{Ax+B}{1+v^2} + \frac{C}{1-v} + \frac{D}{1+v} \quad \cancel{\text{(iii)}}$$

$$\Rightarrow v^3 - 3v = (Av + B)(1 - v^2) + C(1+v)(1+v^2) + D(1-v)(1+v^2)$$

Put $v = 1$

$$-2 = 4C \Rightarrow \boxed{C = -\frac{1}{2}}$$

Put $v = -1$

$$2 = 4D \Rightarrow \boxed{D = \frac{1}{2}}$$

Put $v = 0$

$$0 = B + C + D \Rightarrow \boxed{B = 0}$$

compairing the coefficient v bothside, we get

$$-3 = A + C - D$$

$$\Rightarrow -3 = A - 1$$

$$\Rightarrow \boxed{A = -2}$$

$$\therefore \frac{v^3 - 3v}{(1+v^2)(1-v)(1+v)} = \frac{-2v}{1+v^2} + \frac{-1/2}{1-v} + \frac{1/2}{1+v}$$

From equation (ii)

$$\int \frac{v^3 - 3v}{(1+v^2)(1-v)(1+v)} dv = \int \frac{1}{x} dx$$

$$\Rightarrow \int \left(\frac{-2v}{1+v^2} - \frac{1}{2} \frac{1}{1-v} + \frac{1}{2} \frac{1}{1+v} \right) dv = \int \frac{1}{x} dx$$

$$\Rightarrow -\ln(1+v^2) + \frac{1}{2}\ln(v-1) + \frac{1}{2}\ln|v+1| = \ln|x| + \ln c'$$

$$\Rightarrow \ln \frac{(v^2-1)^{1/2}}{(v^2+1)} = \ln x + \ln c = \ln(xc)$$

$$\Rightarrow \ln \frac{\sqrt{v^2-1}}{v^2+1} = \ln xc$$

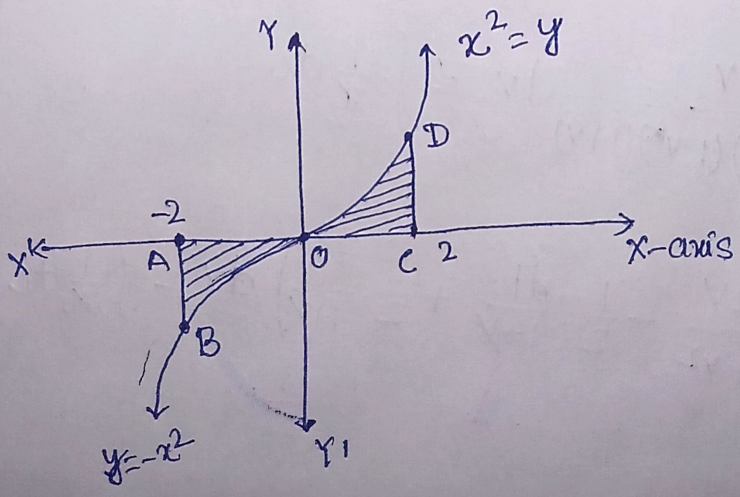
$$\Rightarrow \frac{\sqrt{v^2-1}}{v^2+1} = xc$$

$$\Rightarrow \frac{x\sqrt{y^2-x^2}}{y^2+x^2} = xc$$

$$\Rightarrow \boxed{\frac{\sqrt{y^2-x^2}}{y^2+x^2} = c}$$

33. Given region bounded by curve $y = x|x|$ ~~at~~ x-axis and $x=-2, x=2$

$$\therefore y = \begin{cases} x^2; & x \geq 0 \\ -x^2; & x \leq 0 \end{cases}$$



Area of the shaded region OAB and OCD $=$ ar(OAB) $+$ ar(OCD)

$$= \left| \int_{-2}^{0} x|x|\,dx \right| + \left| \int_{0}^{2} x|x|\,dx \right|$$

$$= \left| \int_{-2}^{0} x|x|\,dx \right| + \left| \int_{0}^{2} x|x|\,dx \right|$$

$$= \left| \int_{-2}^{0} -x^2\,dx \right| + \left| \int_{0}^{2} x^2\,dx \right|$$

$$= \left| -\left(\frac{x^3}{3}\right)_{-2}^{0} \right| + \left| \left(\frac{x^3}{3}\right)_{0}^{2} \right|$$

$$= \left| -\frac{1}{3}\left(0-(-2)^3\right) \right| + \left| \frac{1}{3}\left(2^3-0\right) \right|$$

$$= \frac{8}{3} + \frac{8}{3} = \frac{16}{3}$$

Hence required area is $\frac{16}{3}$ sq. units.

34. By partial fraction

$$\frac{x}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4}$$

$$\Rightarrow x = A(x^2+4) + (Bx+C)(x-1).$$

Put $x=1$ $\quad 1 = 5A \Rightarrow \boxed{A = 1/5}$

Put $x=0$ $\quad 0 = 4A - C \Rightarrow C = 4A \Rightarrow \boxed{C = \frac{4}{5}}$

Compairing both sides, the coefficient of x

$1 = C - B$

$\Rightarrow B = C - 1$

$= \frac{4}{5} - 1$

$\Rightarrow \boxed{B = \frac{-1}{5}}$

$\therefore \frac{x}{(x-1)(x^2+4)} = \frac{1/5}{x-1} + \frac{-\frac{1}{5}x + \frac{4}{5}}{x^2+4}$

$\therefore \int \frac{dx}{(x-1)(x^2+4)} dx = \int \left(\frac{1}{5}\frac{1}{x-1} - \frac{1}{5}\frac{x}{x^2+4} + \frac{4}{5}\frac{1}{x^2+4}\right) dx$

$= \frac{1}{5} \log|x-1| - \frac{1}{10} \log|x^2+4| + \frac{2}{5} \tan^{-1}\left(\frac{x}{2}\right) + c$

$= \log \frac{(x-1)^{1/5}}{(x^2+4)^{1/10}} + \frac{2}{5} \tan^{-1}\left(\frac{x}{2}\right) + c$

$\boxed{34.}$ (b) $= \int_0^1 \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx$

Let $x = \tan\theta$

$\Rightarrow dx = \sec^2\theta \, d\theta$

As $x \to 0 \Rightarrow \theta \to 0$

$x \to 1 \Rightarrow \theta \to \pi/4$

$$\therefore \quad I = \int_0^{\pi/4} \frac{\tan\theta \cdot \tan^{-1}(\tan\theta)}{(1+\tan^2\theta)^{3/2}} \sec^2\theta \, d\theta$$

$$= \int_0^{\pi/4} \frac{\theta \tan\theta}{\sec^3\theta} \cdot \sec^2\theta \, d\theta$$

$$= \int_0^{\pi/4} \theta \cdot \sin\theta \, d\theta$$

$$= \left[\left(\int \sin\theta \, d\theta\right) \theta \right]_0^{\pi/4} - \int_0^{\pi/4} \left[\left(\int \sin\theta \, d\theta\right) \frac{d}{d\theta}(\theta) \, d\theta \right]$$

$$= (-\theta \cos\theta)_0^{\pi/4} + \int_0^{\pi/4} \cos\theta \, d\theta$$

$$= -\frac{\pi}{4} \frac{1}{\sqrt{2}} + (\sin\theta)_0^{\pi/4}$$

$$= -\frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} - 0$$

$$\Rightarrow \boxed{\int_0^1 \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx = \frac{4-\pi}{4\sqrt{2}}}$$

35. Intersection point of two vectors

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j}) = (1+3\lambda)\hat{i} + (1-\lambda)\hat{j} - \hat{k}$$

and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k}) = (4+2\mu)\hat{i} + (-1+3\mu)\hat{k}$

is they are same at that point

i.e., $(1+3\lambda)\hat{i} + (1-\lambda)\hat{j} - \hat{k} = (4+2\mu)\hat{i} + (-1+3\mu)\hat{k}$

$\therefore 1-\lambda=0$

$\Rightarrow \boxed{\lambda=1}$

$-1=-1+3\mu$

$\Rightarrow \boxed{\mu=0}$

$\therefore$ the point is $4\hat{i}-\hat{k}=(4,0,-1)$

Let 'L' be the required ~~vector~~ straight line parallel to

$$\frac{x-1}{-2}=\frac{7-y}{-3}=z$$

So, d.c.s of L is $\langle -2,3,1\rangle$

Hence equation of straight line passing through $(4,0,-1)$ and having d.c.s $\langle -2,3,1\rangle$ is

$$\frac{x-4}{-2}=\frac{y-0}{3}=\frac{z-(-1)}{1}$$

$$\Rightarrow \boxed{\frac{x-4}{-2}=\frac{y}{3}=\frac{z+1}{1}}$$

SECTION-E

36. ATQ, Box-I has 30 red caps.

Box-II has 20 red caps and 10 green caps

and Box-III has 30 green caps.

$P(B_i)=\frac{i}{6}$ (probability of Box-i is selected & a cap is picked)

Here B_1, B_2 and B_3 are three boxes and c is caps.

(i). Denot R = Red cap Selection

G = Green cap Selection

$\therefore P(R/B_1) = \frac{30}{30} = 1$

$P(R/B_2) = \frac{20}{30} = \frac{2}{3}$

$P(R/B_3) = \frac{0}{30} = 0$

By theorem of total probability

$$P(R) = P(B_1)\,P\left(\frac{R}{B_1}\right) + P(B_2)\,P\left(\frac{R}{B_2}\right) + P(B_3)\,P\left(\frac{R}{B_3}\right)$$

$$= \frac{1}{6}(1) + \frac{2}{6}\left(\frac{2}{3}\right) + \frac{3}{6}(0)$$

$$= \frac{1}{6}\left(1+\frac{4}{3}\right) = \frac{7}{18}$$

Hence probability that he selects a red cap $\frac{7}{18}$.

(ii). P(Select a Green cap) $= P(G) = 1 - P(R)$

$$= 1 - \frac{7}{18} = \frac{11}{18}$$

By conditional probability

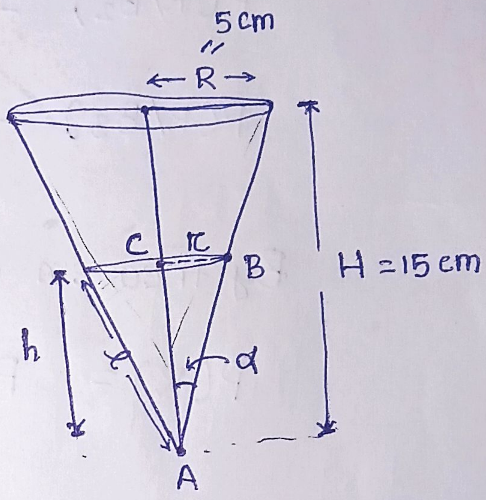
$$\therefore P\left(\frac{B_2}{G}\right) = \frac{P(B_2)\,P\left(\frac{G}{B_2}\right)}{P(G)}$$

$$= \frac{\frac{2}{6} \times \frac{10}{30}}{\frac{11}{18}}$$

$$= \frac{\frac{1}{3} \times \frac{1}{3}}{\frac{11}{18}}$$

$$\Rightarrow \boxed{P\left(\frac{B_2}{G}\right) = \frac{2}{11}}$$

37. In the given cone



(i) Semi-vertical angle = α

$$\Rightarrow \boxed{\angle CAB = \alpha}$$

In the right angle triangle,

$$\tan\alpha = \frac{BC}{AC}$$

$$= \frac{r}{h} = \frac{R}{H} = \frac{5}{15} = \frac{1}{3}$$

$$\Rightarrow \boxed{h = \frac{r}{\tan\alpha}} \text{ or } \boxed{h = 3r}$$

(ii) Rate of being poured = $0.1\ cm^3/s$.

$$\Rightarrow \boxed{\frac{dV}{dt} = 0.1\ cm^3/s.}$$

Here $V = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi \left(\frac{h}{3}\right)^2 h$$

$$\Rightarrow \boxed{V = \frac{1}{27}\pi h^3}$$

$$\Rightarrow \frac{dv}{dt} = \frac{1}{27}\pi 3h^2 \frac{dh}{dt}$$

$$\Rightarrow 0.1 \times \frac{9}{\pi h^2} = \frac{dh}{dt}$$

At depth (h) = 6 cm

$$\left.\frac{dh}{dt}\right|_{h=6} = 0.1 \times \frac{9}{\pi(6^2)} = \frac{1}{40\pi} \text{ cm/s}$$

Hence rate is the juice level in the cup rising when juice is 6 cm deep is $\frac{1}{40\pi}$ cm/s.

(iii) (a) upper surface area $(A_S) = \pi r^2$

$$= \pi\left(\frac{h}{3}\right)^2$$

$$= \frac{1}{9}\pi h^2$$

$$\Rightarrow \frac{dA_S}{dt} = \frac{1}{9}\pi 2h \frac{dh}{dt}$$

At $h = 6$ cm & $\frac{dh}{dt} = \frac{1}{40\pi}$ cm/s

$$\Rightarrow \frac{dA_S}{dt} = \frac{2}{9}\pi(6) \times \frac{1}{40\pi}$$

$$= \frac{1}{3} \times \frac{1}{10}$$

$$\Rightarrow \boxed{\frac{dA_S}{dt} = \frac{1}{30} \text{ cm}^2/\text{s}}$$

(iii) (b) wetted surface area $(C_S) = \pi r l$

$$= \pi \frac{h}{3}\sqrt{h^2 + r^2}$$

$$= \frac{1}{3}\pi h \sqrt{h^2 + \frac{h^2}{9}}$$

$$\Rightarrow C_s = \frac{\sqrt{10}}{9}\pi h^2$$

$$\Rightarrow \frac{dC_s}{dt} = \frac{2\sqrt{10}}{9}\pi h \frac{dh}{dt}$$

At $h = 6\,cm$, $\frac{dh}{dt} = \frac{1}{40\pi}$ cm/s

$$\therefore \frac{dC_s}{dt} = \frac{2\sqrt{10}}{9}\pi (6) \frac{1}{40\pi}$$

$$= \frac{1}{3}\frac{\sqrt{10}}{10}\pi$$

$$\Rightarrow \boxed{\frac{dC_s}{dt} = \frac{\pi}{3\sqrt{10}}\ cm^2/s}$$

Hence wetted surface area increasing at the rate $\frac{\pi}{3\sqrt{10}}$ cm²/s.

38. (i) Let wooden box length = l cm

breadth = b cm

and height = h cm

ATQ,

$l + b = 3 + h \Rightarrow l + b - h = 3$ —— (i)

$2l + 3b + h = 10$ —— (ii)

$b + 7h = 3l - 1 \Rightarrow 3l - b - 7h = 1$ —— (iii)

Matrix representation of the above system of equation is given by

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix} \begin{bmatrix} l \\ b \\ h \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

$\Rightarrow AX = B$

$\Rightarrow X = A^{-1}B$ ———— (iv) ; if $\det(A) \neq 0$.

Where $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}$, $\begin{bmatrix} l \\ b \\ h \end{bmatrix} = X$ and $B = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$

(ii) Here $|A| = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{vmatrix}$

expansion through 1st row

$= 1(-21+1) - 1(-14-3) - 1(-2-9)$

$= -20 + 17 + 11$

$= \cancel{14}\,8$

$\therefore A^{-1}$ exist.

(iii) (a) Cofactor $C_{ij} = (-1)^{i+j} M_{ij}$

$C_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 1 \\ -1 & -7 \end{vmatrix} = -21 + 1 = -20$

$C_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 1 \\ 3 & -7 \end{vmatrix} = -(-14-3) = 17$

$C_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} = -2-9 = -11$

$C_{21} = (-1)^{2+1} \begin{vmatrix} 1 & -1 \\ -1 & -7 \end{vmatrix} = -(-7-1) = 8$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & -1 \\ 3 & -7 \end{vmatrix} = -7+3 = -4$$

$$C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} = -(-1-3) = 4$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} = 1+3 = 4$$

$$C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = -(1+2) = -3$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 3-2 = 1$$

$$\therefore \operatorname{adj}(A) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

$$= \begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \operatorname{adj}(A) = \frac{1}{8} \begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}$$

(iii) (b) $$A^2 = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 7 \\ 11 & 10 & -6 \\ -20 & 7 & 45 \end{bmatrix}$$

$$\therefore A^2 + 7I = \begin{bmatrix} 0 & 5 & 7 \\ 11 & 10 & -6 \\ -20 & 7 & 45 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 5 & 7 \\ 11 & 17 & -6 \\ -20 & 7 & 52 \end{bmatrix}$$