SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks : $+3$ If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

Q.1 Let $\mathbb{R}$ denote the set of all real numbers. Let $a_i, b_i \in \mathbb{R}$ for $i \in \{1, 2, 3\}$.
Define the functions $f: \mathbb{R} \to \mathbb{R}$, $g: \mathbb{R} \to \mathbb{R}$, and $h: \mathbb{R} \to \mathbb{R}$ by

$$f(x) = a_1 + 10x + a_2x^2 + a_3x^3 + x^4,$$
$$g(x) = b_1 + 3x + b_2x^2 + b_3x^3 + x^4,$$
$$h(x) = f(x+1) - g(x+2).$$

If $f(x) \neq g(x)$ for every $x \in \mathbb{R}$, then the coefficient of x^3 in $h(x)$ is

(A)	8
(B)	2
(C)	-4
(D)	-6

Q.2 Three students S_1, S_2, and S_3 are given a problem to solve. Consider the following events:

U: At least one of S_1, S_2, and S_3 can solve the problem,
V: S_1 can solve the problem, given that neither S_2 nor S_3 can solve the problem,
W: S_2 can solve the problem and S_3 cannot solve the problem,
T: S_3 can solve the problem.

For any event E, let $P(E)$ denote the probability of E. If

$$P(U) = \frac{1}{2}, \quad P(V) = \frac{1}{10}, \quad \text{and} \quad P(W) = \frac{1}{12},$$

then $P(T)$ is equal to

(A)	$\frac{13}{36}$	(B)	$\frac{1}{3}$	(C)	$\frac{19}{60}$	(D)	$\frac{1}{4}$

Q.3 Let $\mathbb{R}$ denote the set of all real numbers. Define the function $f: \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} 2 - 2x^2 - x^2 \sin\frac{1}{x} & \text{if } x \neq 0, \\ 2 & \text{if } x = 0. \end{cases}$$

Then which one of the following statements is TRUE?

(A)	The function f is **NOT** differentiable at $x = 0$
(B)	There is a positive real number δ, such that f is a decreasing function on the interval $(0, \delta)$
(C)	For any positive real number δ, the function f is **NOT** an increasing function on the interval $(-\delta, 0)$
(D)	$x = 0$ is a point of local minima of f

Q.4 Consider the matrix

$$P = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Let the transpose of a matrix X be denoted by X^T. Then the number of 3×3 invertible matrices Q with integer entries, such that

$$Q^{-1} = Q^T \text{ and } PQ = QP,$$

is

(A)	32	(B)	8	(C)	16	(D)	24

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks : $+4$ **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : $+3$ If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : $+2$ If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks : $+1$ If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get $+4$ marks;
choosing ONLY (A) and (B) will get $+2$ marks;
choosing ONLY (A) and (D) will get $+2$ marks;
choosing ONLY (B) and (D) will get $+2$ marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of options will get -2 marks.

Q.5 Let L_1 be the line of intersection of the planes given by the equations

$$2x + 3y + z = 4 \text{ and } x + 2y + z = 5 .$$

Let L_2 be the line passing through the point $P(2, -1, 3)$ and parallel to L_1. Let M denote the plane given by the equation

$$2x + y - 2z = 6 .$$

Suppose that the line L_2 meets the plane M at the point Q. Let R be the foot of the perpendicular drawn from P to the plane M .

Then which of the following statements is (are) TRUE?

(A)	The length of the line segment PQ is $9\sqrt{3}$
(B)	The length of the line segment QR is 15
(C)	The area of ΔPQR is $\frac{3}{2}\sqrt{234}$
(D)	The acute angle between the line segments PQ and PR is $\cos^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

Q.6 Let ℕ denote the set of all natural numbers, and ℤ denote the set of all integers. Consider the functions $f: \mathbb{N} \to \mathbb{Z}$ and $g: \mathbb{Z} \to \mathbb{N}$ defined by

$$f(n) = \begin{cases} (n+1)/2 & \text{if } n \text{ is odd}, \\ (4-n)/2 & \text{if } n \text{ is even}, \end{cases}$$

and

$$g(n) = \begin{cases} 3+2n & \text{if } n \geq 0, \\ -2n & \text{if } n < 0. \end{cases}$$

Define $(g \circ f)(n) = g(f(n))$ for all $n \in \mathbb{N}$, and $(f \circ g)(n) = f(g(n))$ for all $n \in \mathbb{Z}$.

Then which of the following statements is (are) TRUE?

(A)	$g \circ f$ is **NOT** one-one and $g \circ f$ is **NOT** onto
(B)	$f \circ g$ is **NOT** one-one but $f \circ g$ is onto
(C)	g is one-one and g is onto
(D)	f is **NOT** one-one but f is onto

Q.7 Let ℝ denote the set of all real numbers. Let $z_1 = 1 + 2i$ and $z_2 = 3i$ be two complex numbers, where $i = \sqrt{-1}$. Let

$$S = \{(x, y) \in \mathbb{R} \times \mathbb{R} : |x + iy - z_1| = 2|x + iy - z_2|\}.$$

Then which of the following statements is (are) TRUE?

(A)	S is a circle with centre $\left(-\frac{1}{3}, \frac{10}{3}\right)$
(B)	S is a circle with centre $\left(\frac{1}{3}, \frac{8}{3}\right)$
(C)	S is a circle with radius $\frac{\sqrt{2}}{3}$
(D)	S is a circle with radius $\frac{2\sqrt{2}}{3}$

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:

  Full Marks : +4 If ONLY the correct numerical value is entered in the designated place;

  Zero Marks : 0 In all other cases.

Q.8 Let the set of all relations R on the set $\{a, b, c, d, e, f\}$, such that R is reflexive and symmetric, and R contains exactly 10 elements, be denoted by $\mathcal{S}$.

Then the number of elements in $\mathcal{S}$ is __________________.

Q.9 For any two points M and N in the XY-plane, let $\overrightarrow{MN}$ denote the vector from M to N, and $\vec{0}$ denote the zero vector. Let P, Q and R be three distinct points in the XY-plane. Let S be a point inside the triangle ΔPQR such that

$$\overrightarrow{SP} + 5\,\overrightarrow{SQ} + 6\,\overrightarrow{SR} = \vec{0}.$$

Let E and F be the mid-points of the sides PR and QR, respectively. Then the value of

$$\frac{\text{length of the line segment } EF}{\text{length of the line segment } ES}$$

is ________________.

Q.10 Let S be the set of all seven-digit numbers that can be formed using the digits 0, 1 and 2. For example, 2210222 is in S, but 0210222 is **NOT** in S.
Then the number of elements x in S such that at least one of the digits 0 and 1 appears exactly twice in x, is equal to _____________.

Q.11 Let α and β be the real numbers such that

$$\lim_{x\to 0} \frac{1}{x^3}\left(\frac{\alpha}{2}\int_0^x \frac{1}{1-t^2}\,dt + \beta x\cos x\right) = 2\,.$$

Then the value of $\alpha + \beta$ is ___________.

Q.12 Let $\mathbb{R}$ denote the set of all real numbers. Let $f: \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) > 0$ for all $x \in \mathbb{R}$, and $f(x+y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$.

Let the real numbers $a_1, a_2, \dots, a_{50}$ be in an arithmetic progression. If $f(a_{31}) = 64f(a_{25})$, and

$$\sum_{i=1}^{50} f(a_i) = 3(2^{25}+1)\,,$$

then the value of

$$\sum_{i=6}^{30} f(a_i)$$

is ___________________.

Q.13 For all $x > 0$, let $y_1(x)$, $y_2(x)$, and $y_3(x)$ be the functions satisfying

$$\frac{dy_1}{dx} - (\sin x)^2\, y_1 = 0, \quad y_1(1) = 5\,,$$
$$\frac{dy_2}{dx} - (\cos x)^2\, y_2 = 0, \quad y_2(1) = \frac{1}{3}\,,$$
$$\frac{dy_3}{dx} - \left(\frac{2-x^3}{x^3}\right) y_3 = 0, \quad y_3(1) = \frac{3}{5e}\,,$$

respectively. Then

$$\lim_{x\to 0^+} \frac{y_1(x)y_2(x)y_3(x) + 2x}{e^{3x}\sin x}$$

is equal to _______________.

SECTION 4 (Maximum Marks: 12)

- This section contains **THREE (03)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks : $+4$ **ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

Q.14 Consider the following frequency distribution:

Value	4	5	8	9	6	12	11
Frequency	5	f_1	f_2	2	1	1	3

Suppose that the sum of the frequencies is 19 and the median of this frequency distribution is 6.

For the given frequency distribution, let α denote the mean deviation about the mean, β denote the mean deviation about the median, and σ^2 denote the variance.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

List-I	List-II
(P) $7f_1 + 9f_2$ is equal to	(1) 146
(Q) 19α is equal to	(2) 47
	(3) 48
(R) 19β is equal to	(4) 145
(S) $19\sigma^2$ is equal to	(5) 55

(A)	(P) → (5)	(Q) → (3)	(R) → (2)	(S) → (4)
(B)	(P) → (5)	(Q) → (2)	(R) → (3)	(S) → (1)
(C)	(P) → (5)	(Q) → (3)	(R) → (2)	(S) → (1)
(D)	(P) → (3)	(Q) → (2)	(R) → (5)	(S) → (4)

Q.15 Let $\mathbb{R}$ denote the set of all real numbers. For a real number x, let $[x]$ denote the greatest integer less than or equal to x. Let n denote a natural number.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

List-I	List-II
(P) The minimum value of n for which the function $f(x) = \left[\frac{10x^3 - 45x^2 + 60x + 35}{n}\right]$ is continuous on the interval $[1, 2]$, is	(1) 8
	(2) 9
(Q) The minimum value of n for which $g(x) = (2n^2 - 13n - 15)(x^3 + 3x)$, $x \in \mathbb{R}$, is an increasing function on $\mathbb{R}$, is	
(R) The smallest natural number n which is greater than 5, such that $x = 3$ is a point of local minima of $h(x) = (x^2 - 9)^n(x^2 + 2x + 3)$, is	(3) 5
(S) Number of $x_0 \in \mathbb{R}$ such that $l(x) = \sum_{k=0}^{4}\left(\sin\lvert x - k\rvert + \cos\left\lvert x - k + \frac{1}{2}\right\rvert\right)$, $x \in \mathbb{R}$, is **NOT** differentiable at x_0, is	(4) 6
	(5) 10

(A)	(P) ⟶ (1) (Q) ⟶ (3) (R) ⟶ (2) (S) ⟶ (5)
(B)	(P) ⟶ (2) (Q) ⟶ (1) (R) ⟶ (4) (S) ⟶ (3)
(C)	(P) ⟶ (5) (Q) ⟶ (1) (R) ⟶ (4) (S) ⟶ (3)
(D)	(P) ⟶ (2) (Q) ⟶ (3) (R) ⟶ (1) (S) ⟶ (5)

Q.16 Let $\vec{w} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$, and $\vec{u}$ and $\vec{v}$ be two vectors, such that $\vec{u} \times \vec{v} = \vec{w}$ and $\vec{v} \times \vec{w} = \vec{u}$. Let α, β, γ, and t be real numbers such that

$$\vec{u} = \alpha\hat{\imath} + \beta\hat{\jmath} + \gamma\hat{k}, \quad -t\,\alpha + \beta + \gamma = 0, \quad \alpha - t\,\beta + \gamma = 0, \quad \text{and } \alpha + \beta - t\,\gamma = 0.$$

Match each entry in List-I to the correct entry in List-II and choose the correct option.

List-I	**List-II**
(P) $\|\vec{v}\|^2$ is equal to	(1) 0
(Q) If $\alpha = \sqrt{3}$, then γ^2 is equal to	(2) 1
(R) If $\alpha = \sqrt{3}$, then $(\beta + \gamma)^2$ is equal to	(3) 2
(S) If $\alpha = \sqrt{2}$, then $t + 3$ is equal to	(4) 3
	(5) 5

(A)	(P) ⟶ (2)	(Q) ⟶ (1)	(R) ⟶ (4)	(S) ⟶ (5)
(B)	(P) ⟶ (2)	(Q) ⟶ (4)	(R) ⟶ (3)	(S) ⟶ (5)
(C)	(P) ⟶ (2)	(Q) ⟶ (1)	(R) ⟶ (4)	(S) ⟶ (3)
(D)	(P) ⟶ (5)	(Q) ⟶ (4)	(R) ⟶ (1)	(S) ⟶ (3)