Standard Mathematics (041)

Class-X ; Pre-Board

Section-A

1. (a) 0

2. (b) $ax^2 + c = 0$; $a > 0$, $c < 0$

3. (d) 10

4. (a) 2

5. (a) $\left(\frac{7}{2}, \frac{11}{4}\right)$

6. (d) 10 cm

7. (d) $3 \cot\alpha : 2$

8. (c) 14

9. (b) 6

10.

11. (c) 60°

12. (b) $72\ cm^2$

13. (a) $90\ cm^2$

14. (c) NOT

15. (d) 8 cm

16. (b) 10-15

17. (b) 1/9

18. (b) $\frac{3-2\sqrt{3}}{3+2\sqrt{3}}$

19. (a) Both 'A' and 'R' are true and 'R' is the correct explanation of 'A'

20. (c) 'A' is true but 'R' is false.

Section-B

21.

$0.2x + 0.3y = 1.3$ ——— (i)

$0.4x + 0.5y = 2.3$ ——— (ii)

Eqn(i) × 2 : $0.4x + 0.6y = 2.6$

Eqn(ii) × 1 : $0.4x + 0.5y = 2.3$

(−)

$0.1y = 0.3$ [1]

$\Rightarrow \boxed{y = 3}$

From (i) $0.2x + 0.3(3) = 1.3$

$\Rightarrow 0.2x = 1.3 = 0.9$

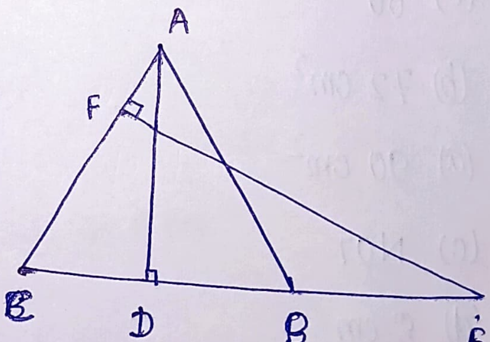
$\Rightarrow x = \dfrac{0.4}{0.2}$ [1]

$\Rightarrow \boxed{x = 2}$

22.

In a isosceles ΔABC, $AB = AC$

$EF \perp AC$ and $AD \perp BC$.



In ΔADB and ΔEFC

$\angle ADB = \angle EFC$ (given)

$\angle B = \angle C$ (isoscele)

$\therefore$ By AA criterion

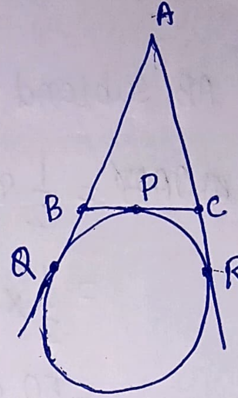
$\Delta ADB \sim \Delta EFC$ [1]

$\Rightarrow \dfrac{AB}{CE} = \dfrac{AD}{EF} = \dfrac{BD}{CF}$

Consider $\dfrac{AB}{EC} = \dfrac{AD}{EF}$ [1]

$\Rightarrow AB \times EF = AD \times EC$

[23.] Given, CR and BQ are tangent to the circle at R and Q respectively.



So $BQ = BP$ and $CP = CR$ ---- [1]

Again AR and AQ are tangent.

So $AR = AQ$

Perimeter of ΔABC

$= AB + BC + AC$

$= AB + BP + PC + CA$

$= AB + BQ + CR + AC$

$= AQ + AR$ ---- [1]

$= 2AR$

$= 2(15)$ (given $AR = 15$ cm)

$= 30$ cm.

[24.] In a clock hour hand = 3 cm

$\Rightarrow$ radius $(r) = 3$ cm.

Angle made by hour clock from 8 am to 9 am

$= 5 \times 6^\circ$

$= 30^\circ$ (As 1 second $= 6^\circ$) [1]

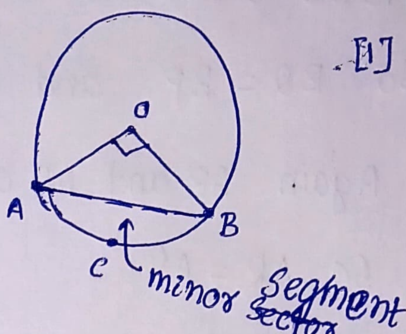
$\therefore$ Area cover by it $= \frac{\theta}{360^\circ} \times \pi r^2$ [1]

$= \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 3^2$

$= \frac{33}{14} \approx 2.35$ cm^2.

24. OR

Given AB subtend angle 90°. and radius $= 10$ cm



Area of $\triangle OAB = \frac{1}{2} r^2 \sin\theta$ [1]

$= \frac{1}{2} \times 10^2 \times \sin 90^\circ$

$= 50 \text{ cm}^2$

Area of minor segment

$=$ Area of minor sector $-$ Area of $\triangle OAB$

$= \frac{\theta}{360^\circ} \times \pi r^2 - 50$ [1]

$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times 10^2 - 50$

$= \frac{11}{7} \times 50 - 50 = 50 \times \frac{4}{7}$

$= \frac{200}{7} \approx 28.56 \text{ cm}^2$

25. Given $7\sin^2\theta + \cos^2\theta = 4$

$\Rightarrow 7\tan^2\theta + 1 = 4\sec^2\theta$

$\Rightarrow 7\tan^2\theta + 1 = 4 + 4\tan^2\theta$ [1]

$\Rightarrow 3\tan^2\theta = 3$

$\Rightarrow \tan\theta = \pm 1$

$\Rightarrow \boxed{\theta = \frac{\pi}{4}}$ [1]

25.OR Given $3x = \operatorname{cosec}\theta$ ~~$\Rightarrow x = \frac{1}{3}$~~

~~$\Rightarrow 9x^2 = \operatorname{cosec}^2\theta$~~

and $\frac{3}{x} = \cot\theta$

So $3\left(x^2 - \frac{1}{x^2}\right)$

$= \frac{1}{3}\left((3x)^2 - \left(\frac{3}{x}\right)^2\right)$

$= \frac{1}{3}\left(\operatorname{cosec}^2\theta - \cot^2\theta\right) = \frac{1}{3}$

Section-C

26. ~~Suppose that $(\sqrt{2}+\sqrt{3})^2$ is irrational.~~

Consider $(\sqrt{2}+\sqrt{3})^2 = 2+3+2\sqrt{6}$

$= 5+2\sqrt{6}$

Suppose that $(\sqrt{2}+\sqrt{3})^2$ is irrational

then $(\sqrt{2}+\sqrt{3})^2 = \frac{p}{q}$; where p and q are coprime [1]

i.e. HCF(p,q) = 1

$\Rightarrow 5+2\sqrt{6} = \frac{p^2}{q^2}$ [1]

$\Rightarrow 2\sqrt{6} = \frac{p^2-5q^2}{2q^2}$

Since $\frac{p^2-5q^2}{2q^2}$ is rational but given $\sqrt{6}$ is irrational [1]

which is a contradiction

Hence $(\sqrt{2}+\sqrt{3})^2$ is irrational.

27.

Given α and β are zeroes of $3x^2-5x+1$

So $\alpha+\beta = \frac{5}{3}$ and $\alpha\beta = \frac{1}{3}$ [1]

quadratic polynomial whose roots are 3α and 3β are

$$K\{x^2-(3\alpha+3\beta)x + 3\alpha\cdot 3\beta\} \; ; K \neq 0$$ [1]

$$= K\{x^2-3\left(\tfrac{5}{3}\right)x + 9\left(\tfrac{1}{3}\right)\} \; ; K \neq 0$$ [1]

$$= K(x^2-5x+9) \; ; K \neq 0.$$

28.

Let monthly hostel charge $= a$

~~Hostel ch~~ food cost per day $= d$

ACQ, $a + 22d = 1380$ ——— (i) [½]

$a + 28d = 1680$ ——— (ii) [½]

$Eq^n(ii) - Eq^n(i)$: $6d = 300$

$\Rightarrow \boxed{d = 50}$ [½]

From (i) $a = 1380 - 22(50)$

$= 1380 - 1100$

$\Rightarrow \boxed{a = 280}$ [½]

∴ Fixed charges is ₹280 and food cost per day is ₹50

28. OR

Let bag contains 5 rupees coins $= x$ [½]

and 2 rupees coins $= y$ [½]

ACQ, $x + y = 94$ ——— (i) [½]

$5x + 2y = 308$ ——— (ii) [½]

~~$\Rightarrow x + 2($~~

$Eq^n(ii) \times 1 : 5x + 2y = 308$

$Eq^n(i) \times 2 : 2x + 2y = 188$

(−)

$3x = 120$

$\Rightarrow \boxed{x = 40}$ [½]

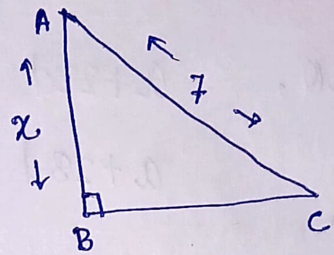
[½]

From (i) $y = 94 - 40 \Rightarrow \boxed{y = 54}$

Hence no. of 5 rupees coins are 40 and no of 2 rupees coins are 54.

29. Given in ΔABC, $AB = x$ units, $AC = 7$ units and $\cos B = 0$
$\Rightarrow \boxed{B = 90^\circ}$ [1/2]

$\therefore BC = \sqrt{AC^2 - AB^2} = \sqrt{7^2 - x^2}$ [1/2]



$\tan C = \dfrac{AB}{BC} = \dfrac{x}{\sqrt{7^2-x^2}}$ [1/2]

$\cot A = \dfrac{AB}{BC} = \dfrac{x}{\sqrt{7^2-x^2}}$

$\cos A = \dfrac{AB}{AC} = \dfrac{x}{7}$ [1/2]

$\sin C = \dfrac{AB}{AC} = \dfrac{x}{7}$

$\cos B = 0$ (given)

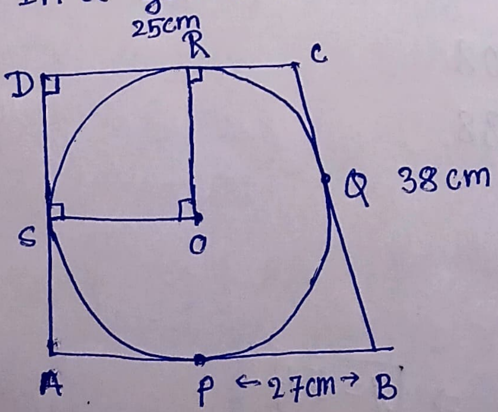
Consider $\sqrt{7-x}\tan C + \sqrt{7+x}\cot A - 14\cos A + 21\sin C + \sqrt{49+x^2}\cos B$

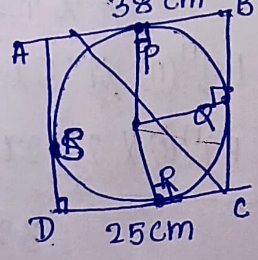
$= \sqrt{7-x} \cdot \dfrac{x}{\sqrt{7^2-x^2}} + \sqrt{7+x}\,\dfrac{x}{\sqrt{7^2-x^2}} - 14\,\dfrac{x}{7} + 21\,\dfrac{x}{7} + \sqrt{49+x^2} \times (0)$ [1/2]

$= \dfrac{x}{\sqrt{7+x}} + \dfrac{x}{\sqrt{7-x}} - 2x + 3x$

$= x + \dfrac{x}{\sqrt{7+x}} + \dfrac{x}{\sqrt{7-x}}$

30. In a given ~~recto~~quadrilateral





Here AB, BC, CD and DA are tangent at P, Q, R and S respectively.

Clearly $OS \perp DS$ and $OR \perp DC$.

In OSDR quadrilateral all angles are 90°.

$\Rightarrow$ OSDR is a rectangle.

Since $DR = DS$ (tangent drawn from an external point D to the circle length are equal)

$\Rightarrow$ OSDR is a square.

$\Rightarrow OS = OR = DS = DR$

As $PB = BQ = 27\,cm$

$\therefore CQ = BC - BQ$

$= 38 - 27 \Rightarrow \boxed{CQ = 11\ cm}$

$\Rightarrow \boxed{RC = CQ = 11\ cm}$

$\Rightarrow DR = DC - RC$

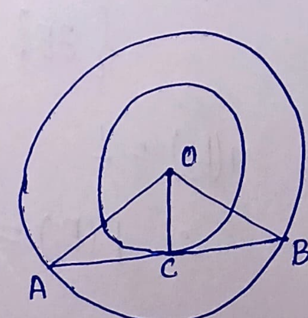
$= 25 - 11$

$\Rightarrow \boxed{DR = 14\ cm}$

Hence radius is 14 cm.

$\boxed{30.\ OR}$

A chord AB of a bigger circle ~~touch~~ is tangent of a smaller circle at C.



Join OC.

Clearly $OC \perp AB$ (As line joining centre to the point of contact of the tangent is $\perp r$).

Join OA and OB.

35. Given

Interval	Frequency (f)	Cumulative Frequency (c.f)
20-40	12	12
40-60	18	30
60-80	23	53 ←
80-100	15	68
100-120	12	80
120-140	12	92
140-160	8	100

[1]

Here $N = \Sigma f = 100 \Rightarrow \frac{N}{2} = 50$

Median class : 60-80 [1]

l = Lower limit of the median class = 60

h = median class length = 20

f = frequency of the median class = 23

c_f = preceeding cumulative frequency of the median class

= ~~53~~ 30

median is given as

$$\text{Median} = l + \frac{\frac{N}{2} - c_f}{f} \times h$$ [1]

$$= 60 + \frac{50 - \cancel{53}\,30}{23} \times 20$$ [1]

$$= 60 - \frac{\cancel{30}\,20}{23} \times 20$$

$$= 60 \times \frac{22}{23} = \frac{1320}{23}$$

[1]

$\approx$ ~~57.87~~ 77.39

Section-D

2. Let no of students $= x$

Contribution of each student $= ₹\, y$

[1]

ACQ, $xy = 2000$ ——— (i)

If 5 student ~~foot~~ not attend and contribution increased by ₹20

[1]

then $(y+20)(x-5) = 2000$ ——— (ii)

From (i) and (ii)

$xy + 20x - 5y - 100 = xy$

$\Rightarrow 20x - 5y = 100$

$\Rightarrow 4x - y = 20$ ——— (iii)

[1/2]

From (i) $xy = 2000$

[1/2]

$\Rightarrow x(4x-20) = 2000$

$\Rightarrow x^2 - 5x = \cancel{400}\ 500$

$\Rightarrow x(x-5) = 20 \times 25$

[1/2]

$\Rightarrow \boxed{x = 25}$

From (iii) $y = 4x - 20$

$= 4(25) - 20$

[1/2]

$\Rightarrow \boxed{y = 80}$

Hence no. of students attend the picnic is 25 and each contribute ~~₹80~~ ₹100

[1]

32. OR

Let A finish a work in x days [1/2]

B " " y days [1/2]

ATQ, $x = y - 6$ ——(i) [1]

in 1 day A finish $\frac{1}{x}$ part of whole work [1/2]

in 1 day B finish $\frac{1}{y}$ part of whole work [1/2]

in 1 day both A and B finish $\left(\frac{1}{x}+\frac{1}{y}\right)$ part of whole [1/2]

$\Rightarrow$ in 4 day both A and B finish $4\left(\frac{1}{x}+\frac{1}{y}\right)$ = whole work. [1/2]

$\Rightarrow \frac{xy}{x+y} = 4$ [1/2]

$\Rightarrow xy = 4(x+y)$

$\Rightarrow x(x+6) = 4x + 4(x+6)$ [1/2]

$\Rightarrow x^2 - 8x + 6x - 24 = 0$

$\Rightarrow x(x-2) = 6 \times 4$

$\Rightarrow \boxed{x = 6}$

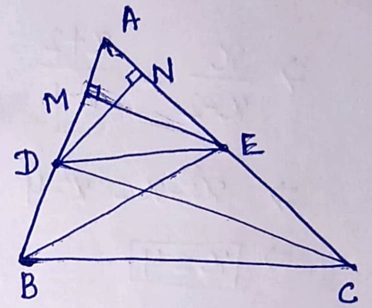
From(i) $y = x + 6 \Rightarrow y = 6 + 6$

$\Rightarrow \boxed{y = 12}$

$\therefore$ Time taken by A to finish the work in 6 days.

33. (Basic Proportionality theorem)

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.



Proof:

Let DE ∥ BE in a ΔABC.

where D and E are points in AB & AC respectively.

Join DC and BE

Draw EM ⊥ AD and DN ⊥ AE

To prove: $\frac{AD}{BD} = \frac{AE}{CE}$

Since $ar(\Delta ADE) = \frac{1}{2} \times EM \times AD$ ——— (i)

$ar(\Delta BDE) = \frac{1}{2} \times BD \times ME$ ——— (ii)

$ar(\Delta ADE) = \frac{1}{2} \times AE \times DN$ ——— (iii)

$ar(\Delta CDE) = \frac{1}{2} \times EC \times DN$ ——— (iv)

Since DE ∥ BC and ΔBDE and ΔCED share same base and height

So $ar(\Delta BDE) = ar(\Delta CED)$

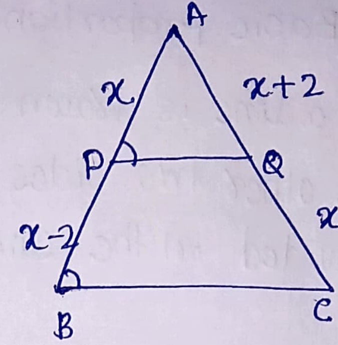
Consider $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{ar(\Delta ADE)}{ar(\Delta CED)}$

$\Rightarrow \frac{\frac{1}{2} \times EM \times AD}{\frac{1}{2} \times BD \times ME} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN}$

$\Rightarrow \boxed{\frac{AD}{BD} = \frac{AE}{CE}}$

Given PQ || BC

By thales theorem



$$\frac{AP}{BP} = \frac{AQ}{CQ}$$

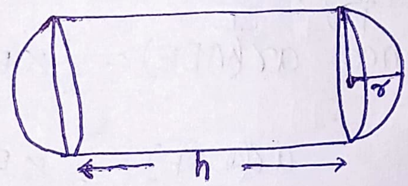
$$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$$

$$\Rightarrow x^2 - x = x^2 - 4$$

$$\Rightarrow \boxed{x = 4}$$

$$\Rightarrow 3x - 5 = 3(4) - 5 = 7$$

34. A gas cylinder with two semi-spherical ends with radius $= r$ and height $= h$.



total length of the gas cylinder $= h + 2r$

$$\Rightarrow 19 = h + 2r$$

$$\Rightarrow h = 19 - d \quad (\text{AS } 2r = d \text{ (diameter)})$$

$$\Rightarrow h = 19 - 7$$

$$\Rightarrow \boxed{h = 12\,m.} \quad \text{and } r = \frac{d}{2} \Rightarrow \boxed{r = \frac{7}{2}\,m.}$$

Surface area of the gas cylinder

= curved Surface area of cylinder + 2(CSA of hemisphere)

$$= 2\pi r h + 2(2\pi r^2)$$

$$= 2\pi r(h + 2r)$$

$$= 2 \times \frac{22}{7} \times \frac{7}{2}\left(12 + 2\left(\frac{7}{2}\right)\right)$$

$$= 22 \times 19$$

$$= 418\ m^2.$$

Volume of the cylinder $= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2 (3h + 4r)$

$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times \left(3(12) + 4\left(\frac{7}{2}\right)\right)$

$= \frac{11}{3} \times \frac{7}{2} \times 50$

$= \frac{1}{3} \times 77 \times 25$

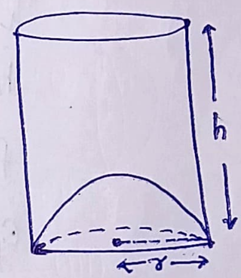
$\approx 641.67 \text{ m}^3$

34. OR

Height of the glass $(h) = 10$ cm

Diameter of the cylinder $= 5$cm

$\Rightarrow$ radius of the cylinder $(r) = \frac{5}{2}$ cm



From the cylinder a hemispherical shape is carved out.

Volume of the apparent capacity $= \pi r^2 h$

$= \frac{22}{7} \times \left(\frac{5}{2}\right)^2 \times 10$

$= \frac{25}{7} \times 11 \times 5 = \frac{1375}{7} \approx 196.43 \text{ cm}^3$

Volume of the actual capacity $= \pi r^2 h - \frac{2}{3}\pi r^3$

$= \pi r^2 \left(h - \frac{2}{3}r\right)$

$= \frac{22}{7} \times \left(\frac{5}{2}\right)^2 \times \left(10 - \frac{2}{3} \times \frac{5}{2}\right)$

$= \frac{11}{7} \times \frac{25}{2} \times \frac{25}{3} = \frac{6875}{42} \approx 163.69 \text{ cm}^3.$

35. Given

Interval	Frequency (f)	Cumulative frequency (c.f)
20-40	12	12
40-60	18	30
60-80	23	53 ←
80-100	15	68
100-120	12	80
120-140	12	92
140-160	8	100

Here $N = \Sigma f = 100 \Rightarrow \frac{N}{2} = 50$

Median class : 60-80

l = Lower limit of the median class = 60

h = median class length = 20

f = frequency of the median class = 23

c_f = preceeding cumulative frequency of the median class = 53

median is given as

$$\text{Median} = l + \frac{\frac{N}{2} - c_f}{f} \times h$$

$$= 60 + \frac{50-53}{23} \times 20$$

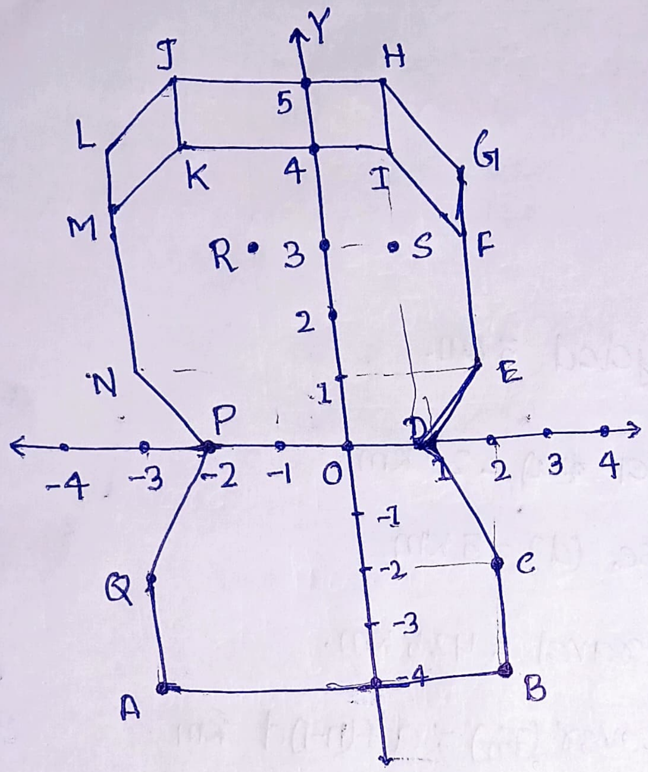
$$= 60 - \frac{60}{23}$$

$$= 60 \times \frac{22}{23} = \frac{1320}{23}$$

$$\simeq 57.87$$

SECTION-E

36.



(i) $R = (-1, 3)$ and $S = (1, 3)$

(ii) width of MKIFGHJL is $5 - 4 = 1$ units

(iii) AB cuts y-axis at $(0, -4)$

OR

KI cuts the y-axis at $(0, 4)$.

37. (i) Let 1st day priya cycled $(a) = 8$ km.

increase the distance each day $(d) = 3$ km

$\therefore$ distance covered in 5^{th} day is $= a + (5-1)d$

$= 8 + 4(3) = 20$ km.

(ii) Let 1st day priya cycled $= a'$ km

each day it increased by d' km

Then ~~in nth day~~ ATQ, $a' + 2d' = 11$ ——— (i)

and $a' + 7d' = 26$ ——— (ii)

From (ii) and (i) $5d' = 15$

$\Rightarrow \boxed{d' = 3}$

From (i) $a + 2d' = 11$

$\Rightarrow a = 11 - 2(3)$

$\Rightarrow \boxed{a = 5}$

So, on 1st day she cycled 5 km.

(iii) 1st day priya travel (a) = 20 km

Each day increase (d) = 5 km

total distance travel = 425 km.

On nth day she cover $(T_n) = a + (n-1)d$ km

$= 20 + (n-1)5$

$= 15 + 5n$ km.

ATQ, $425 = 15 + 5n$

$\Rightarrow 410 = 5n$

$\Rightarrow \boxed{n = 82}$

On 82 days she will reach the destination

OR

Total distance to cover 450 km.

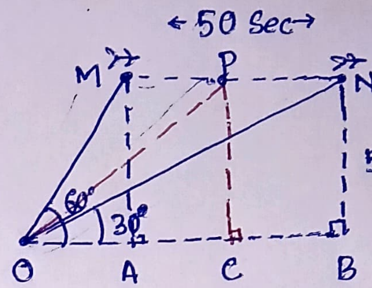
1st day priya travel (a) = 15 km

each day increase (d) = 4 km

In 8 ~~on 8th~~ days she ~~travel~~ covered = $a + 7d$ km

$= 15 + 7(4)$

$= 43$ km.



(i) $\angle MOA = 60^\circ$; $\angle NOB = 30^\circ$

$MA = NB = 500\sqrt{3}$

[1]

(ii) height of the plane from ground $= BN = MA = 500\sqrt{3}$ m.

$\tan 30^\circ = \frac{NB}{OB}$ (In $\triangle OBN$)

$\Rightarrow OB = \frac{500\sqrt{3}}{1/\sqrt{3}} = 15000$m.

In $\triangle OAM$; $\tan 60^\circ = \frac{MA}{OA}$

$\Rightarrow OA = \frac{500\sqrt{3}}{\sqrt{3}} = 5000$m.

$\therefore$ AB = distance between plane position

$= OB - OA = 15000 - 5000$

$\Rightarrow \boxed{AB = 10000\text{m}}$

$\Rightarrow \boxed{MN = 10000\text{m}}$

[1/2]

$\therefore$ Speed of the plane $= \frac{\text{distance}}{\text{time}} = \frac{10000\text{m}}{50 \text{ sec}}$

$= \frac{10\text{km}}{\left(\frac{50}{3600}\right)\text{hr}} = \frac{10 \times 3600}{50}$ km/hr.

[1/2]

$= 720$ km/hr.

(iii) $AB = OB - OA = 15000 - 5000 = 10000$m or 10km.

[1]

OR

After 30 second, angle of elevation is 45°.

$\angle POC = 45^\circ \Rightarrow OC = PC = 5000\sqrt{3}$ m.

$\therefore A \to P = AC = OC - OA = 5000\sqrt{3} - 5000 = 5000(\sqrt{3} - 1)$ m. [1]