SHORT NOTES CLASS : 11 MATH

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1. SETS

The theory of sets was developed by German mathematician Georg cantor (1845-1918).

SET:

A Set is a well-defined collection of objects. Sets usually denoted by capital letters, A, B, C, X, Y, Z etc. It's elements are represented by small letters a, b, c, x, y, z.

$\mathbb{N}$: the set of natural numbers $= \{1, 2, 3, \ldots\}$

$\mathbb{Z}$: $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$ (Integers)

$\mathbb{Q}$: $\{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\}$ (Rational numbers)

$\mathbb{R}$: $(-\infty, +\infty)$ (Real numbers)

$\mathbb{Z}^+$: positive integers $= \{1, 2, \ldots\}$

$\mathbb{Z}^-$: negative integers $= \{-1, -2, \ldots\}$

$\mathbb{Q}^+$: positive rational numbers $= \{\frac{p}{q} : p, q \in \mathbb{Z}^+\}$

$\mathbb{Q}^-$: negative rational numbers $= \{\frac{p}{q} : p, q \in \mathbb{Z}^-\}$

$\mathbb{R}^+$: $(0, \infty)$ (positive real numbers)

W: $\{0, 1, 2, \ldots\}$ (whole numbers)

There are two methods of representing a set.

(i) Roster or tabular form. : listing all elements.

(ii) Set-builder form. : Define with the help of common properties.

Ex:

$A = \{1, 4, 9, 16, 25, \dots\}$ (Roster form)

$A = \{n^2 : n \in \mathbb{N}\}$ (Set-builder form)

Types of set

1. Empty set:

A set which doesn't contain any element is called the empty set or the null set or the void set. It is denoted by ϕ or $\{\}$.

2. Finite and Infinite set:

A set which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

Equal Sets: Two sets A and B are said to be equal if they exactly the same elements and we write $A = B$. Otherwise the sets are said to be unequal and we write $A \neq B$.

Subsets: A set A is said to be a subset of a set B if every element of A is also an element of B. We write $A \subset B$.

Interval:

(i) open interval : $(a, b) = \{x : a < x < b\}$

(ii) closed interval : $[a, b] = \{x : a \leq x \leq b\}$

(iii) Semi-closed or Semi-open interval

$[a, b) = \{x : a \leq x < b\}$

$(a, b] = \{x : a < x \leq b\}$

N.B. length of the interval $[a, b]$, (a, b), $[a, b)$ or $(a, b]$ is $b - a$.

Power set:

The collection of all subsets of a set A is called the power set of A. It is denoted by P(A).

$\boxed{\text{N.B}}$ Number of elements in power set of A is $|P(A)| = 2^{|A|}$.

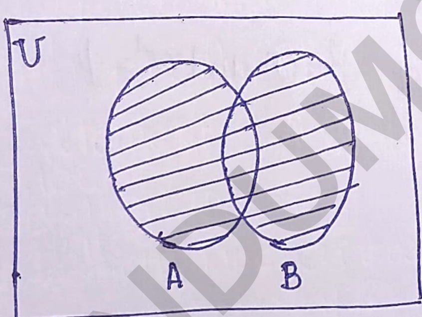
i.e., If $|A| = m$ then $|P(A)| = 2^m$.

Union of Sets:

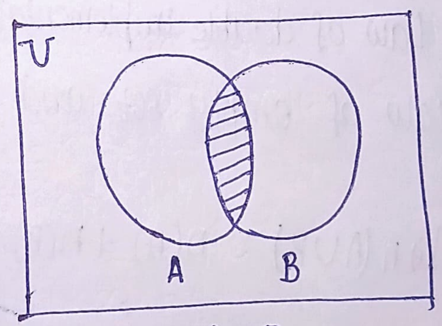
The union of two sets A and B ~~is the Se~~ is denoted and defined as $A \cup B = \{x : x \in A \text{ or } x \in B\}$

Inter Section of Sets:

The intersection of A and B is denoted and defined as $A \cap B = \{x : x \in A \text{ and } x \in B\}$.



$A \cup B$



$A \cap B$

(i) $A \cup B = B \cup A$ (Commutative law)
$A \cap B = B \cap A$

(ii) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)
$(A \cap B) \cap C = A \cap (B \cap C)$

(iii) $A \cup \phi = A$
$A \cap \phi = \phi$

(iv) $A \cup U = U$
$A \cap U = A$

(v) $A \cap A = A$
$A \cup A = A$.

Difference of Sets:

The difference of the sets A and B is denoted and defined as

$$A-B = \{ x : x \in A \text{ and } x \notin B \}$$

Complement of a set

Let U be the universal set and A a subset of U.

Then the complement of A is denoted and defined as

$$U-A = \bar{A} \text{ or } A^c \text{ or } A' = \{ x : x \in U \text{ and } x \notin A \}$$

Properties:

(i) Complement laws: $A \cup A' = U$ and $A \cap A' = \phi$

(ii) Demorgan's law: $(A \cup B)' = A' \cap B'$

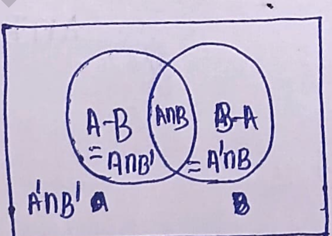
$(A \cap B)' = A' \cup B'$

(iii) Law of double implementation: $(A')' = A$

(iv) Law of empty set and Universal set $\phi' = U$ and $U' = \phi$

$\boxed{\text{N.B.}}$ (i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

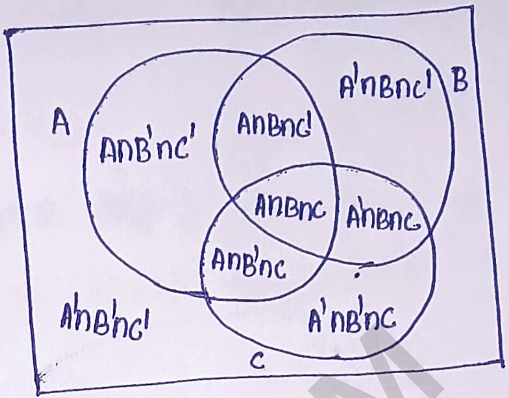
(ii)



(iii) $\max(0, |A|+|B|-|U|) \leq |A \cap B| \leq \min(|A|, |B|)$

$\max(|A|, |B|) \leq |A \cup B| \leq \min(|A|+|B|, |U|)$

N.B. Let A, B and C are three subsets of a universal set U.



2. RELATION AND FUNCTION

Cartesian Set

Given two non empty sets P and Q. The cartesian product $P \times Q$ is the set of all ordered pairs of elements from P and Q.

$$P \times Q = \{(p,q) : p \in P, q \in Q\}.$$

Relation:

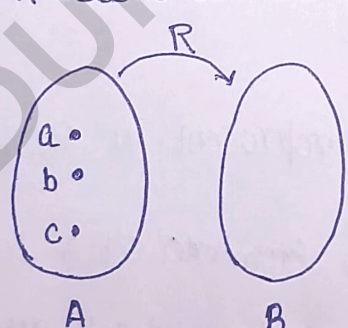
A relation R from a non-empty set A to a non-empty set B is a cartesian product $A \times B$.

$$R \subset A \times B$$

$\boxed{\text{N.B.}}$ (i) R can be empty.

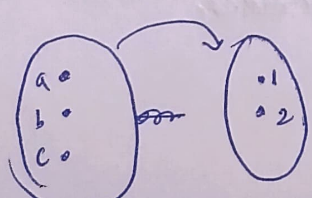
If $R : A \to B$ is a relation then $A \neq \phi$ and $B \neq \phi$.

(ii) ~~If R~~ Second element is called the image of the first element.



R is not a relation as $B \neq \phi$

(iii)



R is a relation as $B \neq \phi, A \neq \phi$

(iv) ϕ is also a relation and called an empty relation

domain of the relation R

The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R.

Range of a Relation R

The set of all second elements in a relation R from a set to a set B is called the range of the relation R.
The whole set B is called the codomain of the relation R.

$\boxed{\text{N.B}}$ (i) range $\subset$ codomain.

Ex: $A = \{1, 2, 3\}$, $B = \{a, b, c\}$

$R = \{(1, a), (2, c)\}$ As $R \neq \phi$.

Here domain of $R = \{1, 2\}$
range of $R = \{a, c\}$
codomain of $R = B$

(ii) Any relation can be represent in Roster form or Set builder form

(iii) Number of Relation from a set A to a set B =
Number of subset on $A \times B = 2^{|A \times B|}$

Functions:

A relation f from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.

In other word Let $A \neq \phi$ and $B \neq \phi$

then $f: A \rightarrow B$ is called a function from A to B if

(i) domain of $f = A$

(ii) $(x, y) \in f$ and $(x, z) \in f$ then $y = z$

Here y is called image of x under f and x is called the preimage of y under f.

Real valued function

A function $f: A \rightarrow B$ is called a real valued function if $B \subset R$ and called a real function if it domain A also in R. i.e. $A \subset R$.

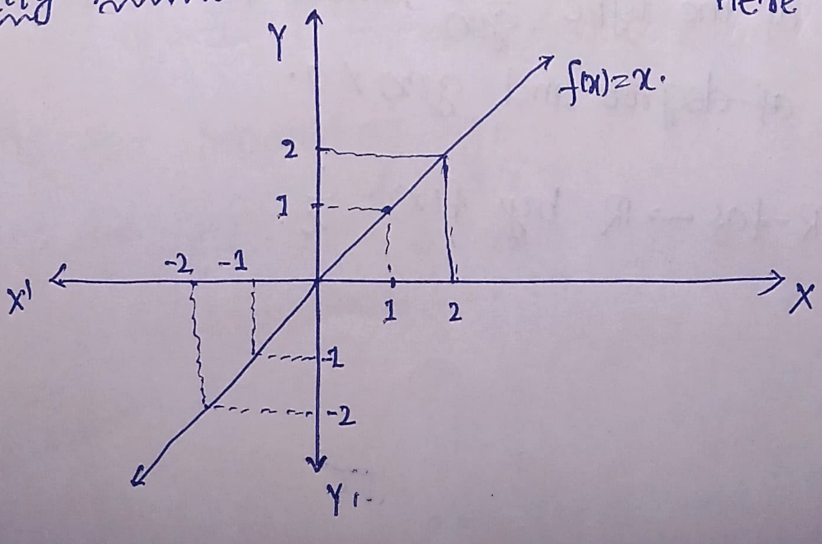
Ex: $f: \mathbb{N} \rightarrow \mathbb{N}$ by $f(x) = 2x+1$

Important function:

(i) Identity function: $f: \mathbb{R} \rightarrow \mathbb{R}$ by $f(x) = x$

Here domain of $f = \mathbb{R}$

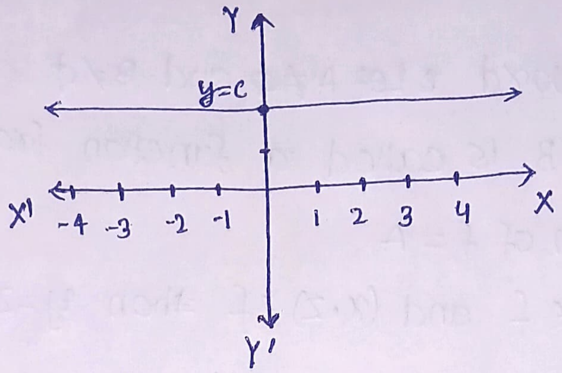
Range of $f = \mathbb{R}$



(ii) **Constant function:**

Define $f: R \to R$ by $f(x) = c$, where $c \in R$ and constant

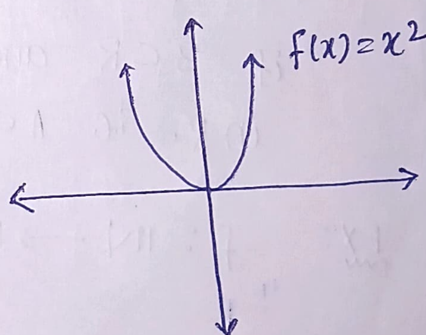
$\text{dom}(f) = R$ and $\text{rng}(f) = \{c\}$



(iii) **Polynomial function:**

A function $f: \mathbb{R} \to \mathbb{R}$ is said to be a polynomial function if for each $x \in \mathbb{R}$, $f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$; where n is a non-negative integer and $a_0, a_1, \dots, a_n \in \mathbb{R}$.

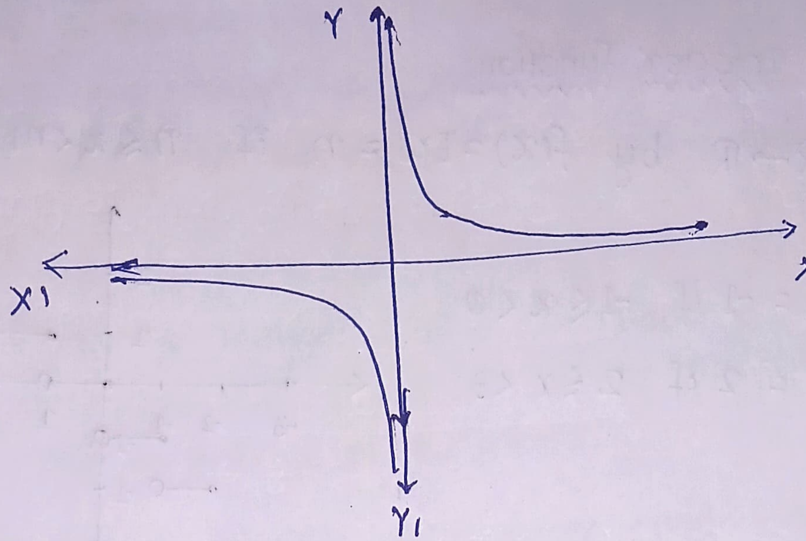
Ex: $f(x) = x^2$
which is a polynomial functions of degree 2.



(iv) **Rational function:**

It is of the type $\frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomial function ~~of degree~~ and $g(x) \neq 0$.

Ex: $f: \mathbb{R} - \{0\} \to \mathbb{R}$ by $f(x) = \frac{1}{x}$

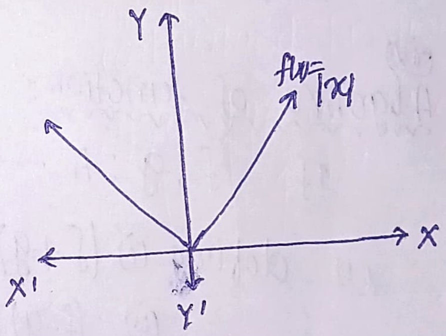


(V) The modulus function:

$f: \mathbb{R} \to \mathbb{R}$ by $f(x) = |x|$

It is also called absolute value function.

It is also can be written as



$$|x| = \begin{cases} x\ ; & x \geq 0 \\ -x\ ; & x < 0 \end{cases}$$

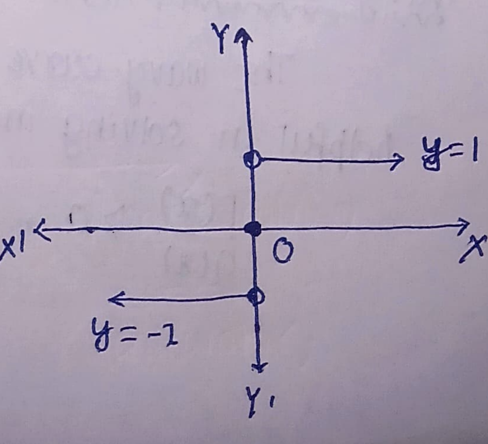
OR $\max(x, -x)$

(VI) Signum function:

$f: \mathbb{R} \to \mathbb{R}$ by $f(x) = \begin{cases} 1\ ; & x > 0 \\ 0\ ; & x = 0 \\ -1\ ; & x < 0 \end{cases}$

It is also denoted as "sgn".

i.e, $\mathrm{sgn}(x) = \begin{cases} 1\ ; & x > 0 \\ 0\ ; & x = 0 \\ -1\ ; & x < 0 \end{cases}$



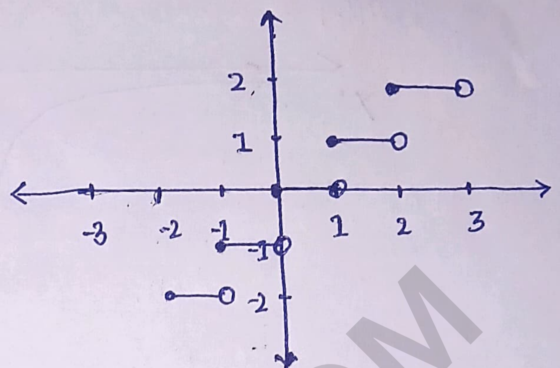
(VII) **Greatest integer function:**

$f : \mathbb{R} \to \mathbb{R}$ by $f(x) = [x] = n$ if $n \leq x < n+1$, where $n \in \mathbb{Z}$

Ex

$[x] = -1$ if $-1 \leq x < 0$

$[x] = 2$ if $2 \leq x < 3$



Properties:

(i) $[x+n] = [x] + n$; $\forall x \in \mathbb{R}$ and $n \in \mathbb{Z}$

(ii) $[-x] = -[x]$, $x \in I$

(iii) $[-x] = -[x] - 1$, $x \notin I$

Algebra of function:

If $f, g : A \longrightarrow B$ are two functions then

define (i) $(f+g)(x) = f(x) + g(x)$

(ii) $(f-g)(x) = f(x) - g(x)$

(iii) $(fg)(x) = f(x)\, g(x)$

(iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

(v) $(Kf)(x) = Kf(x)$

Wavy-curve method:

The wavy curve method or the method of intervals is helpful in solving inequalities of the form:

$\frac{F(x)}{G(x)} > 0$, $\frac{F(x)}{G(x)} \geq 0$, $\frac{F(x)}{G(x)} < 0$ or $\frac{F(x)}{G(x)} \leq 0$.

Step 1: Draw a number line and mark all the critical points. undefined values and unsatisfying values must marked with an empty bubble.

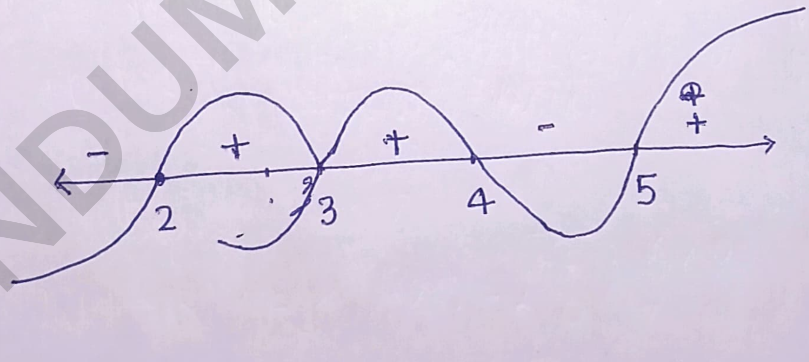
Step-2 Sketch the wavy curve starting from left to right from above the number line if the above step was positive, and below the number line if the above was negative, passing through all the points.

~~(i) If the exponent of the factor is odd, it is called a sample point.~~

~~(ii) If the exponent of the factor is even, it is called a double point.~~

~~Ste~~ (i) If the ~~point is the~~ exponent of the factor is even then the curve located on the same side and if ~~the~~ it is odd then the curve ~~located on the~~ crush the number line.

Example $f(x) = (x-2)(x-3)^2(x-4)(x-5)$

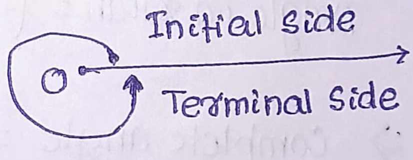


3. TRIGONOMETRIC FUNCTION

The word 'trigonometry' is derived from the greek words 'trigon' and 'metron' and its means 'measuring the sides of a triangle'.

Angles:

Angle is a measure of rotation of a given ray about its initial point. The original ray is called the initial side and the final position of the ray after rotation is called the terminal side of the angle. The point of rotation is called the vertex.



anticlockwise   Clockwise

Degree measure:

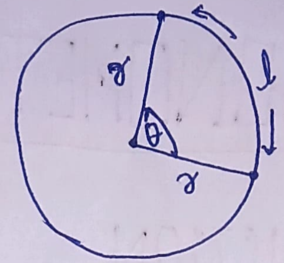
If a rotation from the initial side to terminal side is $\left(\frac{1}{360}\right)^{th}$ of a revolution, the angle is said to have a measure of one degree, written as 1°.

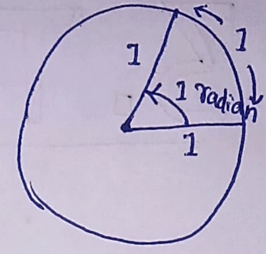
$$1^\circ = 60' \quad (60 \text{ minutes})$$

$$1' = 60'' \quad (60 \text{ Seconds})$$

Radian measure:

Angle subtended at the centre by an arc of length 1 unit in a unit circle is said to have a measure of 1 radian.





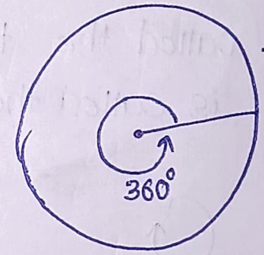
If in a circle of radius r, an arc of length 'l' subtends an angle θ radian at the centre, we have

$$\theta = \frac{l}{r} \text{ radian}$$

Radian symbol used are "rad" or "c".

· Relation between degree and radian:

A circle subtends at the centre an angle whose radian measure is 2π and it degree measure is $360°$.



$$\text{angle in radian} = \left(\frac{\text{Perimeter}}{\text{radius}}\right)$$

$$\Rightarrow \text{Complete angle} = \left(\frac{2\pi r}{r}\right)^c$$

$$\Rightarrow \boxed{360° = (2\pi)^c}$$

or

$\boxed{360 \text{ deg} = 2\pi \text{ rad}}$ or simply $\boxed{360° = 2\pi}$

N.B. Radian measure $= \frac{\pi}{180} \times$ Degree measure

Degree measure $= \frac{180}{\pi} \times$ Radian measure

N-B 1 rad $= 57° \cdot 16' \left(21\frac{9}{11}\right)''$ (Hand calculation)

$\approx 57° \, 16' \, 22''$ or $\boxed{1^c \approx 57.295°}$

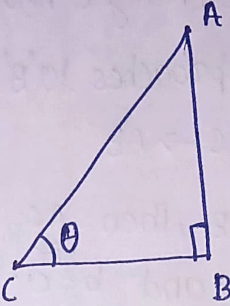
1 rad $\approx 57° \, 17' \, 44''$ (calculator)

Trigonometric function

$\sin\theta = \frac{AB}{AC} \Rightarrow \boxed{\sin\theta = \frac{p}{h}}$

$\cos\theta = \frac{BC}{AC} \Rightarrow \boxed{\cos\theta = \frac{b}{h}}$

$\tan\theta = \frac{AB}{BC} \Rightarrow \boxed{\tan\theta = \frac{p}{b}}$



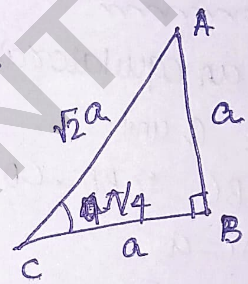
p: perpendicular ; b: base and h: hypotenuse

$\text{cosec}\,\theta = \frac{h}{p}$; $\sec\theta = \frac{h}{b}$ and $\cot\theta = \frac{b}{p}$

For Angle $45^\circ = \pi/4$

Consider an isoscele triangle ABC

$AB = BC = a$ then $AC = \sqrt{2}a$.



so $\sin\frac{\pi}{4} = \frac{AB}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$

$\cos\frac{\pi}{4} = \frac{BC}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$

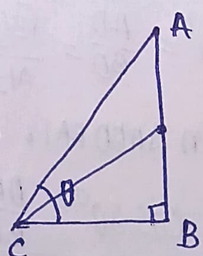
$\tan\frac{\pi}{4} = \frac{AB}{BC} = \frac{a}{a} = 1$

For Angle 0°

since $\sin\theta = \frac{p}{h}$

when $\theta \to 0$ then $p \to 0$ and $h \to b$.

At $\theta = 0$; $p = 0$ and $h = b$.

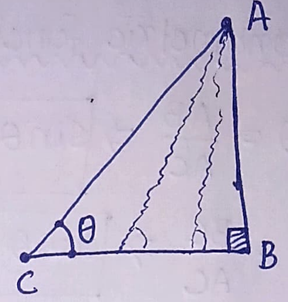


so $\sin 0^\circ = \frac{p}{h} = \frac{0}{b} = 0$

$\cos 0^\circ = \frac{b}{h} = \frac{b}{b} = 1$

$\tan 0^\circ = \frac{p}{b} = \frac{0}{b} = 0$

For angle 90°:



In right angle triangle ABC,

When 'C' point approaches to 'B', then

$CB \to 0$ and $AC \to AB$

ie. When $C = B$, then $AC = AB$ and $BC = 0$

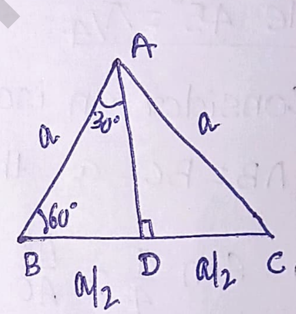
ie. $h = p$ and $b = 0$.

$\therefore \sin 90^\circ = \frac{p}{h} = \frac{p}{p} = 1$

$\cos 90^\circ = \frac{b}{h} = \frac{0}{h} = 0$

$\tan 90^\circ = \frac{p}{b} = \frac{p}{0}$ = undefined

For angle 30° and 60°



Consider an equilateral triangle having side a unit.

As $AD \perp BC \Rightarrow BD = CD = a/2$

$AB = AC = a$

So $AD = \frac{\sqrt{3}}{2} a$

also $\angle BAD = 30^\circ$

In ΔABD RAT, $\sin \angle B = \frac{AD}{AB} = \frac{\sqrt{3}/2\, a}{a} \Rightarrow \boxed{\sin 60^\circ = \frac{\sqrt{3}}{2}}$

$\cos 60^\circ = \frac{BD}{AB} = \frac{a/2}{a} \Rightarrow \boxed{\cos 60^\circ = \frac{1}{2}}$

$\tan 60^\circ = \frac{AD}{BD} = \frac{\sqrt{3}/2\, a}{a/2} \Rightarrow \boxed{\tan 60^\circ = \sqrt{3}}$

again, in ΔABD RAT, $\sin 30^\circ = \frac{BD}{BA} \Rightarrow \boxed{\sin 30^\circ = \frac{1}{2}}$

$\cos 30^\circ = \frac{DA}{BA} = \frac{\sqrt{3}/2\, a}{a} \Rightarrow \boxed{\cos 30^\circ = \sqrt{3}/2}$

$\boxed{\tan 30^\circ = 1/\sqrt{3}}$

Imp

θ	$0°$	$15°$ $\pi/12$	$30°$ $\pi/3$	$45°$ $\pi/4$	$60°$ $\pi/3$	$75°$ $5\pi/12$	$90°$ $\pi/2$
$\sin\theta$	0	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	1
$\cos\theta$	1	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	0
$\tan\theta$	0	$2-\sqrt{3}$	$1/\sqrt{3}$	1	$\sqrt{3}$	$2+\sqrt{3}$	undefined

Imp Properties:

(i)
$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$

(ii)
$$\sin(x+y) = \sin x\cdot\cos y + \cos x\cdot\sin y$$
$$\sin(x-y) = \sin x\cdot\cos y - \cos x\cdot\sin y$$
$$\cos(x+y) = \cos x\cdot\cos y - \sin x\cdot\sin y$$
$$\cos(x-y) = \cos x\cdot\cos y + \sin x\cdot\sin y$$
$$\tan(x+y) = \frac{\tan x + \tan y}{1-\tan x\cdot\tan y}$$
$$\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x\cdot\tan y}$$
$$\cot(x+y) = \frac{\cot x\cdot\cot y - 1}{\cot y + \cot x}$$
$$\cot(x-y) = \frac{\cot x\cdot\cot y + 1}{\cot y - \cot x}$$

(iii)
$$\sin 2x = \begin{cases} 2\sin x\cdot\cos x \\ = \dfrac{2\tan x}{1+\tan^2 x}\end{cases}$$

$$\cos 2x = \begin{cases} = \cos^2 x - \sin^2 x \\ = 2\cos^2 x - 1 \\ = 1 - 2\sin^2 x \\ = \dfrac{1 - \tan^2 x}{1 + \tan^2 x} \end{cases}$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

(iv) $\sin 3x = 3\sin x - 4\sin^3 x$

$\cos 3x = 4\cos^3 x - 3\cos x$

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

(v) $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)$

$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)$

$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x+y}{2}\right)$

$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)$

(vi) $\cos x \cdot \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$

$\sin x \cdot \sin y = -\frac{1}{2}\left[\cos(x+y) - \cos(x-y)\right]$

$\sin x \cdot \cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$

$\cos x \cdot \sin y = \frac{1}{2}\left[\sin(x+y) - \sin(x-y)\right]$

N.B. (i) $\sin x = 0 \Rightarrow x = n\pi \; ; n \in \mathbb{Z}$

$\cos x = 0 \Rightarrow x = (2n+1)\frac{\pi}{2} \; ; n \in \mathbb{Z}$

(ii)

$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$	$\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right) = \sec\theta$
$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$	$\sec\left(\frac{\pi}{2}-\theta\right) = \operatorname{cosec}\theta$
$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$	$\cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$

(iii)

$\sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta$	$\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right) = \sec\theta$
$\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$	$\sec\left(\frac{\pi}{2}+\theta\right) = -\operatorname{cosec}\theta$
$\tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta$	$\cot\left(\frac{\pi}{2}+\theta\right) = -\tan\theta$

(iv)

$\sin(\pi-\theta) = \sin\theta$	$\operatorname{cosec}(\pi-\theta) = \operatorname{cosec}\theta$
$\cos(\pi-\theta) = -\cos\theta$	$\sec(\pi-\theta) = -\sec\theta$
$\tan(\pi-\theta) = -\tan\theta$	$\cot(\pi-\theta) = -\cot\theta$

Imp

N.B. (i) $\sin(n\pi \pm \theta) = \square \sin\theta \; ; \forall n \in \mathbb{Z}$

$\cos(n\pi \pm \theta) = \square \cos\theta \; ; \forall n \in \mathbb{Z}$

$\tan(n\pi \pm \theta) = \square \tan\theta \; ; \forall n \in \mathbb{Z}$

$n\pi$ is called horizontal line

Where $\square$ ← fill with either + or - depending on which quadrant it is.

(ii) $\sin\left((2n+1)\frac{\pi}{2} \pm \theta\right) = \square \cos\theta \; ; \forall n \in \mathbb{Z}$

$\cos\left((2n+1)\frac{\pi}{2} \pm \theta\right) = \square \sin\theta \; ; \forall n \in \mathbb{Z}$

$\tan\left((2n+1)\frac{\pi}{2} + \theta\right) = \square \cot\theta \; ; \forall n \in \mathbb{Z}$

$(2n+1)\frac{\pi}{2}$ is called vertical line

Where $\square$ is fill with + or − depending on which quadrant it is.

Theorem

$\boxed{\text{N.B.}}$ $\cos(\alpha)\cdot\cos(2\alpha)\cdot\cos(2^2\alpha)\cdot\cdots\cos(2^{n-1}\alpha) = \frac{1}{2^n}\frac{\sin(2^n\alpha)}{\sin\alpha}$

$\sin\theta\cdot\sin(60^\circ-\theta)\cdot\sin(60^\circ+\theta) = \frac{1}{4}\sin 3\theta$

$\cos\theta\cdot\cos(60^\circ-\theta)\cos(60^\circ+\theta) = \frac{1}{4}\cos 3\theta$

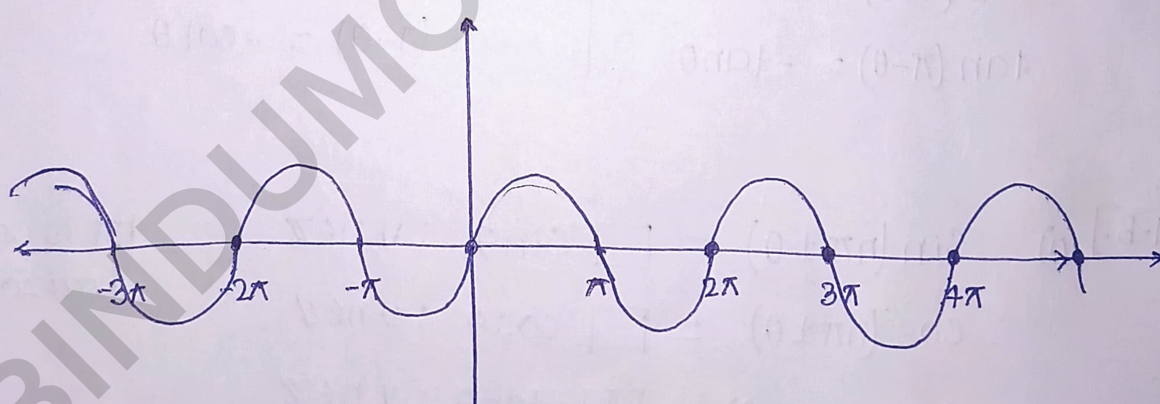
$\tan\theta\cdot\tan(60^\circ-\theta)\tan(60^\circ+\theta) = \tan 3\theta$

Graph of trigonometric function

1. $\sin : \mathbb{R} \to \mathbb{R}$ by $\sin x = \frac{Per}{h}$;

   domain of $\sin x = \mathbb{R}$

   range of $\sin x = [-1, 1]$

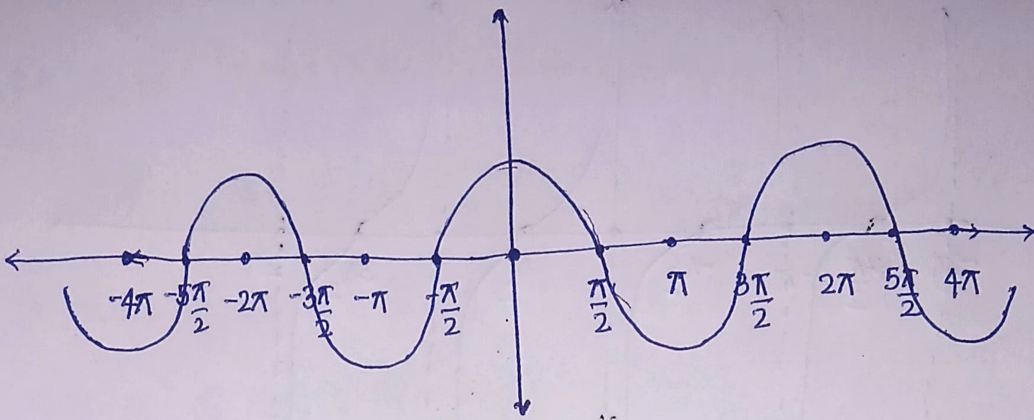


Sinx graph

2. $\cos : \mathbb{R} \to \mathbb{R}$ by $\cos x = \frac{b}{h}$

   domain of $\cos x = \mathbb{R}$

   range of $\cos x = [-1, 1]$



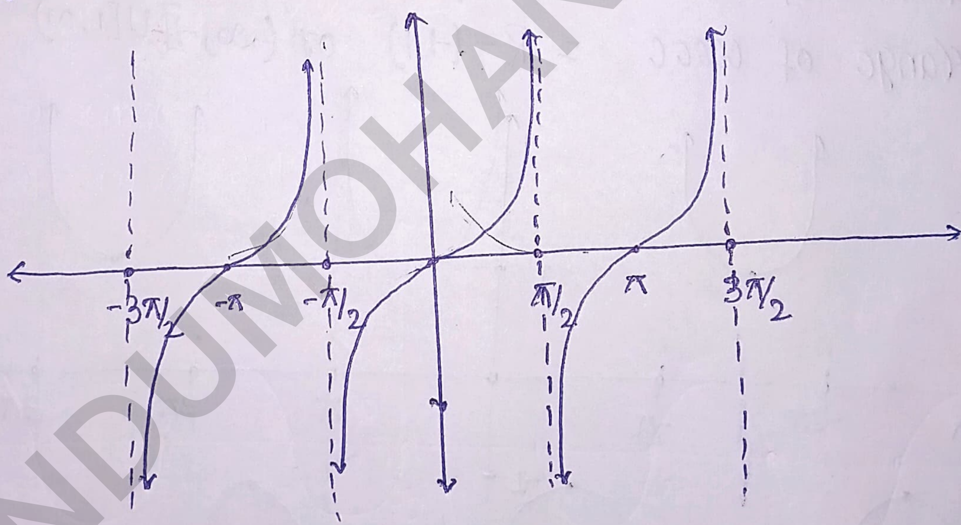
graph of cos x

3. $\tan : \mathbb{R} - \{(2n+1)\frac{\pi}{2} : n \in \mathbb{Z}\} \longrightarrow \mathbb{R}$

by $\tan x = \frac{p}{b}$

domain of $\tan x = \mathbb{R} - \{(2n+1)\frac{\pi}{2} : n \in \mathbb{Z}\}$

range of $\tan x = \mathbb{R} = (-\infty, \infty)$



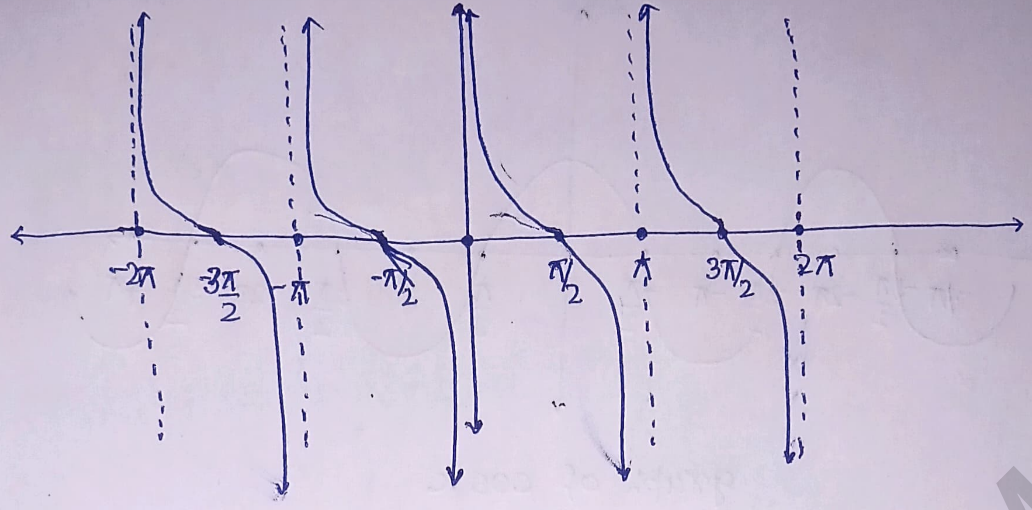
graph of tan x

4. $\cot : \mathbb{R} - \{n\pi : n \in \mathbb{Z}\} \longrightarrow \mathbb{R}$

by $\cot x = \frac{b}{p}$

domain of $\cot x = \mathbb{R} - \{n\pi : n \in \mathbb{Z}\}$

range of $\cot x = \mathbb{R} = (-\infty, +\infty)$



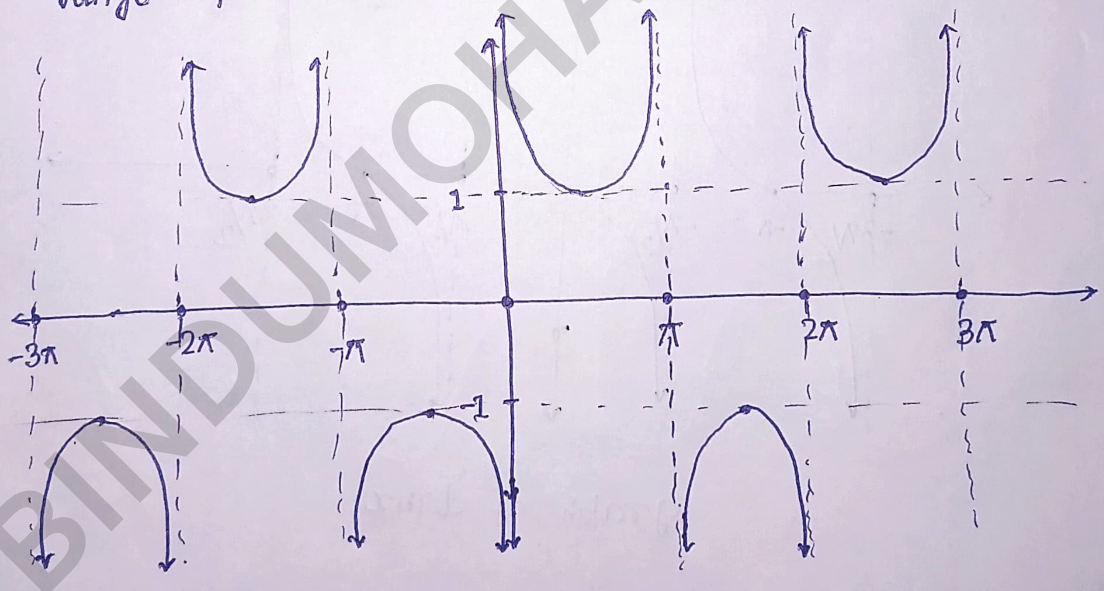
graph of cot x

5. $\operatorname{cosec} : \mathbb{R} - \{n\pi : n \in \mathbb{Z}\} \rightarrow \mathbb{R}$

by $\operatorname{cosec} x = \frac{h}{p}$

domain of cosec = $\mathbb{R} - \{n\pi : n \in \mathbb{Z}\}$

range of cosec $= \mathbb{R} - (-1, 1)$ or $(-\infty, -1] \cup [1, \infty)$



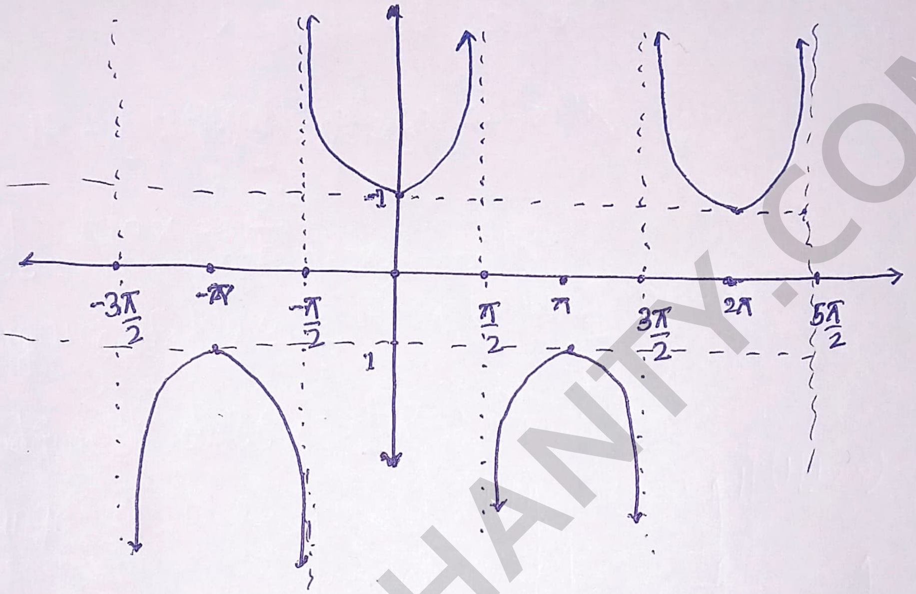
graph of cosec x.

6. $\sec : \mathbb{R} - \{(2n+1)\frac{\pi}{2} : n \in \mathbb{Z}\} \longrightarrow \mathbb{R}$

by $\sec x = \frac{h}{b}$

domain of sec : $\mathbb{R} - \{(2n+1)\frac{\pi}{2} : n \in \mathbb{Z}\}$

range of sec : $\mathbb{R} - (-1,1) = (-\infty,-1] \cup [1,\infty)$



graph of sec x