SHORT NOTES

CLASS : 10

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CHAPTER: 01

$\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$ (Natural numbers)

$W = \{0, 1, 2, 3, 4, 5, \ldots\}$ (Whole numbers)

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ (Integers)

$\mathbb{Q} = \{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\}$ (Rational numbers)

$\mathbb{R} = (-\infty, +\infty)$ (Real number)

Real numbers
- Terminating
- Non-terminating
  - Recurring
  - Non-recurring

(i) Terminating:

Ex: 1.5, 222

(ii) Non-terminating and recurring:

Ex: $0.\overline{3}$, 1.252525...

(iii) Non-terminating and non-recurring:

Ex: 1.253545561..., $\sqrt{2}$, π, e

NOTE: A rational number is said a terminating rational if its denominor is only contains 2 or 5 and its power. i.e., $2^m 5^n$, where m, n are whole numbers.

Division formula:

Dividend = divisor × quotient + Remainder

i.e. $a = bq * r \;;\; 0 \le r < b$

'a' is called divisible by 'b' if $r = 0$ and written as $b \mid a$.

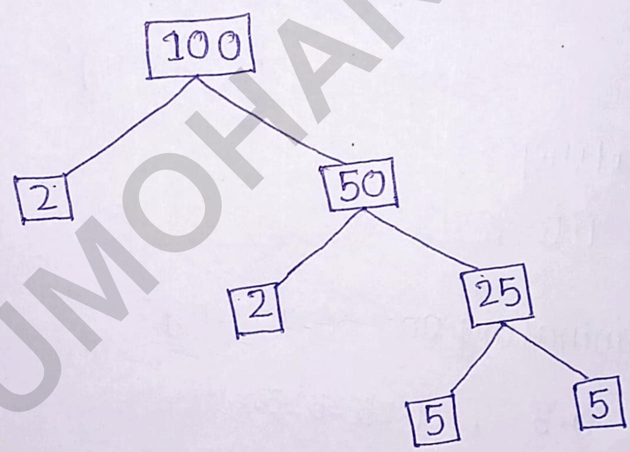
Ex: $3 \mid 6$, $7 \mid 21$

Factor:

A number 'b' is called a factor of 'a' if $b \mid a$.

i.e., $\exists\ m \in \mathbb{Z}$ such that $a = bm$.

Factor tree



$\therefore 100 = 2 \times 2 \times 5 \times 5$

Prime factorization theorem

Any positive number greater then '1' can be written as product of primes and factorisation is unique apart from the in which it occurs.

Ex: $100 = 2^2 \times 5^2$.

Fundamental theorem of arithmetic

Every composite number can be expressed as a product of primes and this factorisation is unique apart from the other in which the prime factors occurs.

Ex: $100 = 2^2 \times 5^2$. (Prime factorisation method)

H.C.F. (Highest common factors)

For any two integers 'a' and 'b' the highest among all common factors is known as H.C.F. of 'a' and 'b'.

Ex: Let $a = p_1^{m_1} p_2^{m_2} \ldots p_k^{m_k}$

and $b = p_1^{n_1} p_2^{n_2} \ldots p_k^{n_k} \ldots p_l^{m_l}$

Then $H.C.F(a,b) = p_1^{\min(m_1,n_1)} p_2^{\min(m_2,n_2)} \ldots p_k^{\min(m_k,n_k)}$

= product of common primes with minimum power.

Ex

$12 = 2^2 \times 3^1$

$18 = 2 \times 3^2$

$\therefore H.C.F.(12,18) = 2^1 \times 3^1 = 6$

L.C.M (Least common multiple)

For any ~~two~~ integers (positive) 'a' and 'b' the lowest among all the common ~~factors~~ multiples is known as L.C.M. of 'a' and 'b'.

Ex: $L.C.M.(a,b) = P_1^{\max(m_1,n_1)} \times \ldots \times P_K^{\max(m_K,n_K)} \times P_{K+1}^{m_{K+1}} \times \ldots \times P_l^{m_l}$

= Product of all primes with its maximum power.

Ex: $LCM(12,18) = 2^2 \times 3^2 = 36$

Relation between H.C.F and L.C.M

1) For any two positive integers

$$H.C.F(a,b) \times L.C.M(a,b) = a \times b$$

2) For any three positive integers:

$$H.C.F.(a,b,c) = \frac{a \times b \times c \times LCM(a,b,c)}{LCM(a,b) \times LCM(b,c) \times LCM(c,a)}$$

$$L.C.M.(a,b,c) = \frac{a \times b \times c \times HCF(a,b,c)}{HCF(a,b) \times HCF(b,c) \times HCF(c,a)}$$

* $\sqrt{2}$, $2\sqrt{2}$, $2-\sqrt{2}$ are all irrational number.
We can prove it by method of contradiction.

NOTE In most of the problems minimum means LCM and maximum means HCF

N.B. Let 'p' be a prime number and $p \mid a^2$ then $p \mid a$, where 'a' is a positive integer.

2. POLYNOMIAL

Polynomial: An ~~equation~~ expression of the form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, where $a_0, a_1, \ldots, a_n$ & $a_n \neq 0$ are real number and x be a variable, is called a polynomial of degree n.

It is denoted as $p(x) = a_n x^n + \cdots + a_1 x + a_0$.

linear polynomial:

A polynomial of degree 1 is called a linear polynomial. It is of the form "$a_1 x + a_0$" where $a_1 \neq 0$, a_1, a_0 are real number.

Quadratic polynomial:

A polynomial of degree 2 is called a quadratic polynomial. It is of the form "$a_2 x^2 + a_1 x + a_0$", where $a_2 \neq 0$, a_0, a_1, a_2 are real number.

cubic polynomial:

A polynomial of degree 3 is called a cubic polynomial. It is of the form "$a_3 x^3 + a_2 x^2 + a_1 x + a$", where $a_3 \neq 0, a_2, a_1, a_0$ are real numbers.

Monomial: A polynomial having only one term.

i.e, of the form $a_n x^n$; a_n is real number and n be any whole number.

Binomial: A polynomial having only two terms.

i.e., of the form $a_n x^n + a_m x^m$; where $n \neq m$, n and m are whole number and a_n, a_m are ~~real~~ non-zero real

Trinomial:

A polynomial having three terms is called a trinomial.

i.e., of the form $a_n x^n + a_m x^m + a_p x^p$; where n, m and p are whole number such that $n \neq m \neq p$ and a_n, a_m, a_p are non-zero real number.

Bi-quadratic polynomial

A polynomial of degree 4 is called a bi-quadratic polynomial.

It is of the form "$a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$" where, $a_4 \neq 0$, a_3, a_2, a_1 & a_0 are real number.

Constant polynomial

A polynomial ~~non~~-of degree zero is called a constant polynomial.

i.e., of the form "kx^0" where k be any real number.

Zero polynomial

An expression "0" is called a zero polynomial.

It's degree is not defined.

$\boxed{\text{N.B.}}$ (i) Every non-zero polynomial are continuous graph.

Zero or Root of a polynomial:

A value of x for which a given polynomial becomes zero is called a zero or root of that polynomial.

$\boxed{\text{N.B.}}$ In your syllabus only real numbers are/can be zero.

A real number 'α' is called a zero of a polynomial $P(x) = a_n x^n + \cdots + a_1 x + a_0$; if $P(\alpha) = 0$.

<u>Ex:</u>

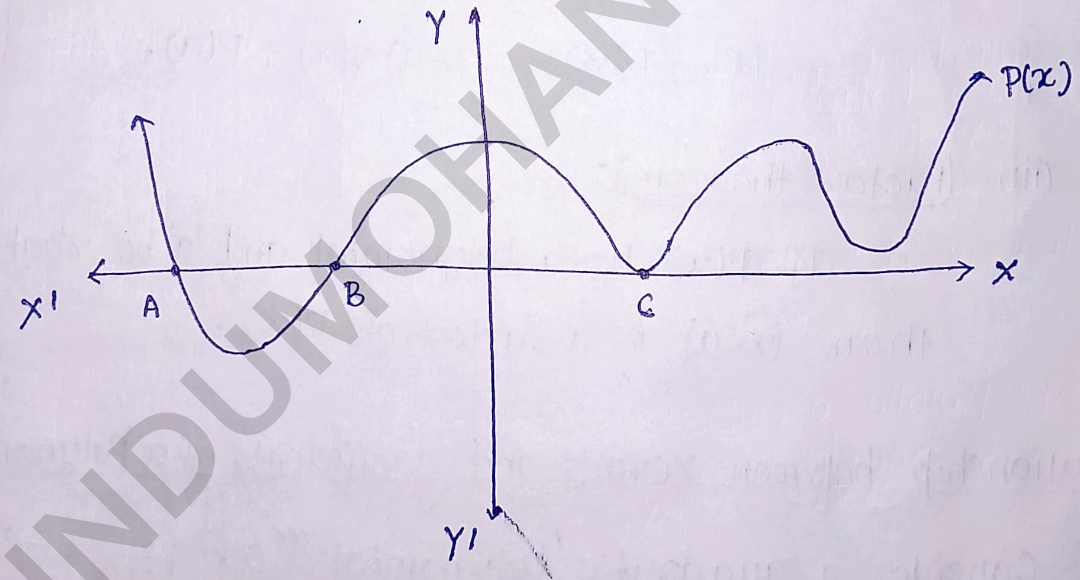
$$P(x) = x^2 - 1$$

Since $P(1) = (1)^2 - 1 = 0$ and $P(-1) = (-1)^2 - 1 = 0$

So, 1 and -1 are zeroes of $P(x)$.

<u>Geometrical representation of zeroes of a polynomial:</u>

The number of times a polynomial intersect with x-axis is the number of zeroes it has and zeroes are those.



Here A, B and C are the zeroes of $P(x)$.

$\boxed{\text{N.B.}}$ Actual number of roots including non-~~zero~~ real number can be greater then it looks.

$\boxed{\text{N.B.}}$ (i) (Integral Root Theorem):

If $P(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$; where $a_{n-1}, \cdots, a_0$ are integers, then any integer root 'α' of $P(x)$ is also a factor of a_0.

(ii) (Rational Root Theorem):

Let $\frac{b}{c}$ be a rational fraction in lowest terms. If $\frac{b}{c}$ is a root of the polynomial $P(x) = a_n x^n + \cdots + a_1 x + a_0$; where $a_n \neq 0, a_{n-1}, \ldots, a_0$ are integers then b is a factor of a_0 and c is a factor of a_n.

N.B. (i) An n^{th} degree polynomial can have at most 'n' real roots.

(ii) (Remainder theorem):

If $P(x)$ be a polynomial and $x-a$ is ~~a fac~~ divided $P(x)$ then remainder obtained is $P(a)$.

i.e., $P(x) = (x-a)Q(x) + P(a)$.

(iii) (Factor theorem):

If $P(x)$ be a polynomial and 'a' be root of $P(x)$ then $(x-a)$ is a factor of $P(x)$.

Relationship between zeroes and coefficients of a polynomial:

(i) Consider a quadratic polynomial "ax^2+bx+c".

Let α and β are two roots.

then $x^2 + \frac{b}{a}x + \frac{c}{a} = (x-\alpha)(x-\beta)$

Comparing bothsides, we get

$\boxed{\alpha+\beta = -\frac{b}{a}}$ and $\Rightarrow$ $\boxed{\text{Sum of zeroes} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}}$

$\boxed{\alpha\beta = \frac{c}{a}}$ $\Rightarrow$ $\boxed{\text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}}$

(ii) Consider a cubic polynomial " ax^3+bx^2+cx+d ".

Let α, β and γ are three roots.

then $x^3+\frac{b}{a}x^2+\frac{c}{a}x+\frac{d}{a} = (x-\alpha)(x-\beta)(x-\gamma)$

On comparing, we get

$\boxed{\alpha+\beta+\gamma = -\frac{b}{a}}$ $\Rightarrow$ $\boxed{\text{Sum of zeroes} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}}$

$\boxed{\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a}}$ $\Rightarrow$ $\boxed{\text{Sum of products of zeroes} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}}$

$\boxed{\alpha\beta\gamma = -\frac{d}{a}}$ $\Rightarrow$ $\boxed{\text{Product of zeroes} = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}}$

NB. The above pattern can be extended to degree-n polynomial.

Ex: Verify $3, -1$ and $-\frac{1}{3}$ are the zeroes of the cubic polynomial $P(x) = 3x^3-5x^2-11x-3$ and then verify the relationship between the zeroes and the coefficients.

Solutions

Given $p(x) = 3x^3-5x^2-11x-3$

Now, $P(3) = 3(3)^3-5(3)^2-11(3)-3 = 0$

$P(-1) = 3(-1)^3-5(-1)^2-11(-1)-3 = 0$

and $P(-\frac{1}{3}) = 3(-\frac{1}{3})^3-5(-\frac{1}{3})^2-11(-\frac{1}{3})-3 =$

Therefore, $3, -1$ and $-\frac{1}{3}$ are zeroes of $P(x)$.

Denote $\alpha = 3$, $\beta = -1$ and $\gamma = -\frac{1}{3}$; $a = 3, b = -5, c = -11$ & $d = -3$

Then $\alpha+\beta+\gamma = 3-1-\frac{1}{3} = \frac{5}{3} = -\frac{b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = 3(-1) + (-1)(-\frac{1}{3}) + (-\frac{1}{3})3 = -\frac{11}{3} = \frac{c}{a}$

and $\alpha\beta\gamma = 3(-1)(-\frac{1}{3}) = 1 = -\frac{d}{a}$.

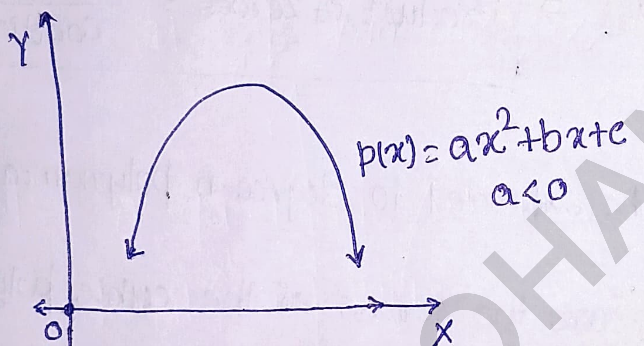
$\boxed{\text{N.B.}}$ An n^{th} degree polynomial can have atmost 'n' real zeroes.

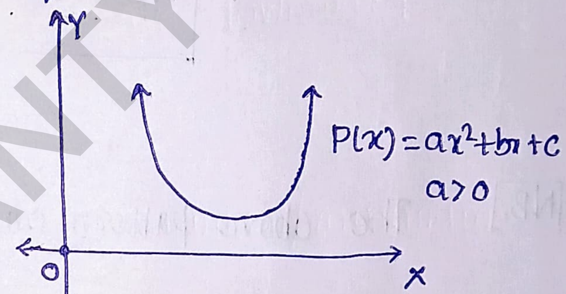
$\boxed{\text{N.B.}}$ (i) Consider a quadratic ~~equation~~polynomial

$p(x) = ax^2 + bx + c$; where $a \neq 0$, b, c are real number. which is represent a parabola.

Case-I

If $a > 0$ then the above ~~equation~~ polynomial is concave upward.





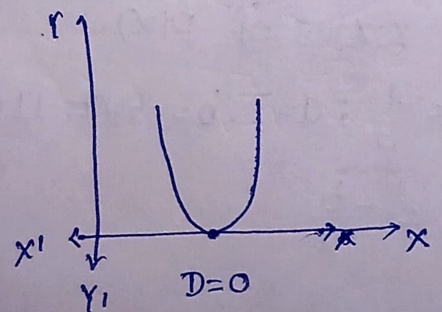
Case-II If $a < 0$, then the above ~~equat~~ polynomial is concave downward

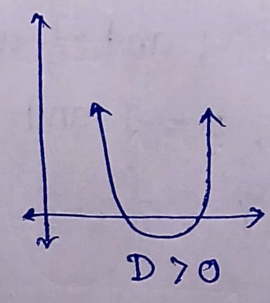
(ii) $D = b^2 - 4ac$ is called discriminant of $p(x) = ax^2 + bx + c; a \neq 0$

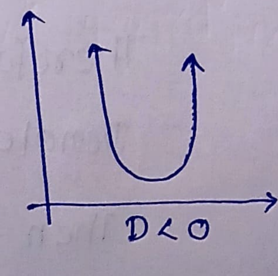
Case-I If $D = 0$, then $p(x) = ax^2 + bx + c$ has ~~only one~~ two same zeroes. (equal real roots)

Case-II If $D > 0$, then $p(x)$ has exactly two zeroes.

Case-III If $D < 0$, then $p(x)$ has no zero.

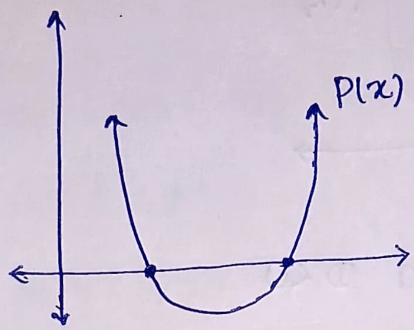






(iii) Consider $P(x) = ax^2 + bx + c$; $a \neq 0$, b, c are real numbers.

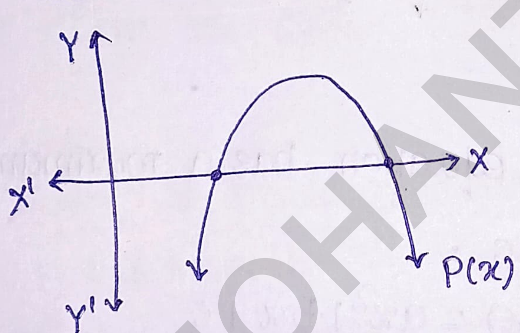
Case-I If $a > 0$ and $D > 0$



It has two zeroes and concave upward.

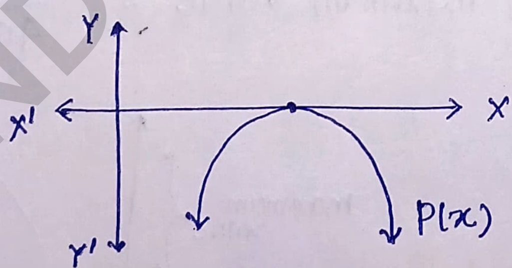
Case-II If $a < 0$ and $D > 0$

It has two zeroes and concave downward



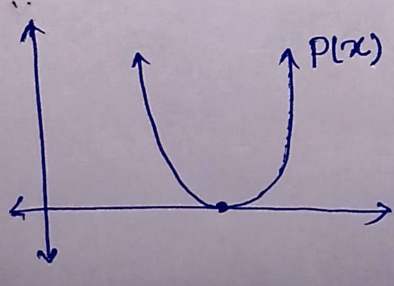
Case-III If $a < 0$ and $D = 0$

It have one zero and concave downward



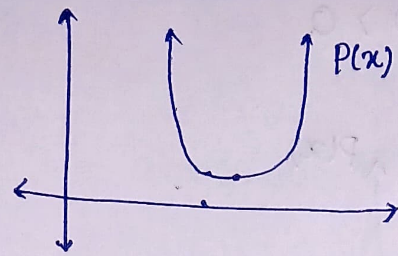
Case-IV If $a > 0$ and $D = 0$

It have one zero and concave upward



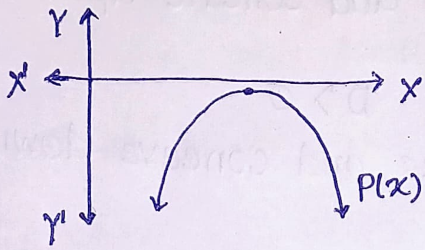
Case-V If $a > 0$ and $D < 0$

It have no zeroes and concave upward



Case-VI If $a < 0$ and $D < 0$

It have no-zeroes and concave downward.

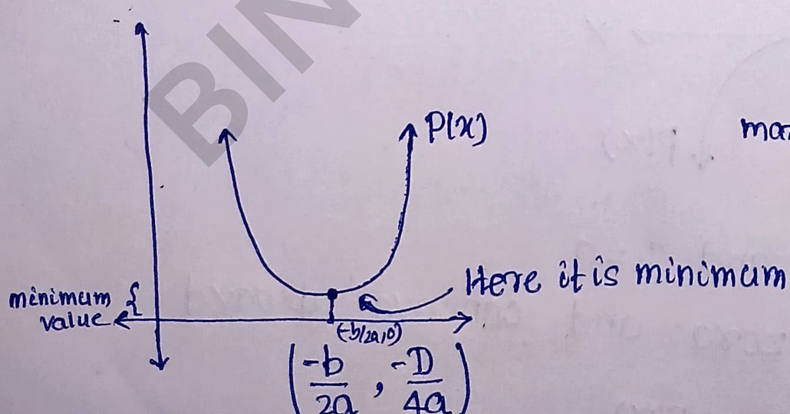


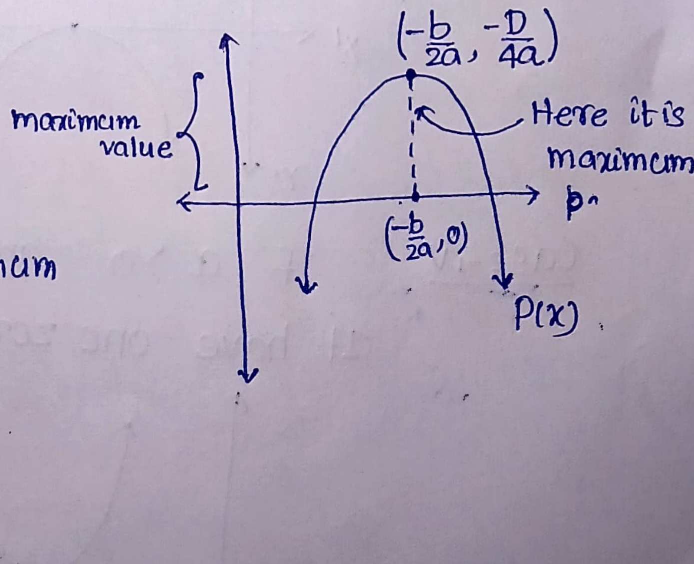
$\boxed{\text{N-B.}}$ A quadratic equation has a maximum/minimum point and value.

For $p(x) = ax^2 + bx + c$

minimum/maximum point $= -\dfrac{b}{2a}$

and minimum/maximum value $= -\dfrac{D}{4a}$





3. Pair Of Linear Equation

Definition:

A pair of linear equations are

$$a_1x + b_1y + c_1 = 0$$

and $a_2x + b_2y + c_2 = 0$

where a_1, b_1, a_2, b_2 are real number

a_1 and b_1 both simultaneously can't be zero

also, a_2 and b_2 both simultaneously can't be zero.

The above are called a pair of system of linear equations.

Conditions:

The pair of linear equations

$$a_1x + b_1y + c_1 = 0 \quad \text{——— (i)}$$

and $a_2x + b_2y + c_2 = 0$ ——— (ii)

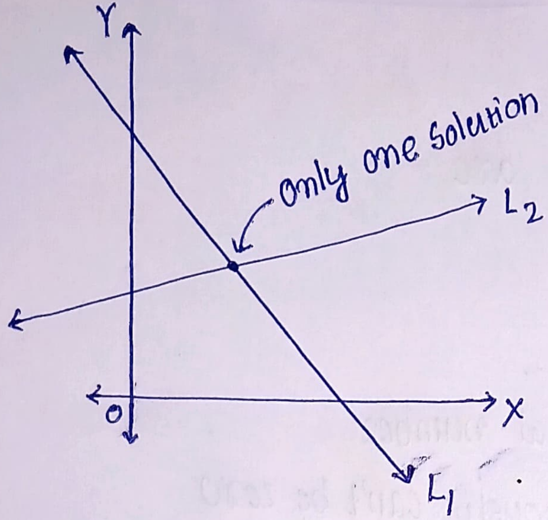
The above pair are

(i) intersecting, iff $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (Consistent and only one solution)

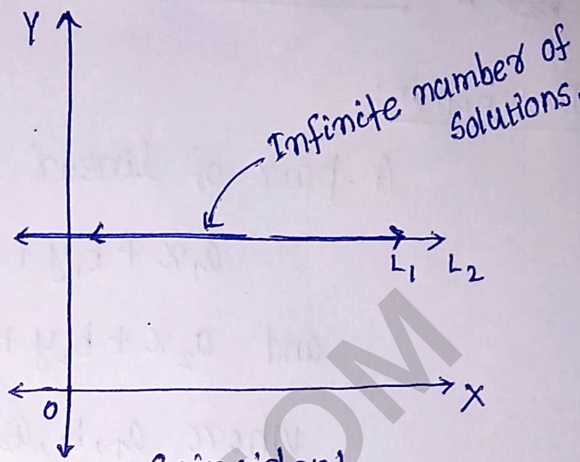
(ii) Coincident iff $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (consistent and infinite number of solutions)

(iii) Parallel iff $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (inconsistent and no solution)

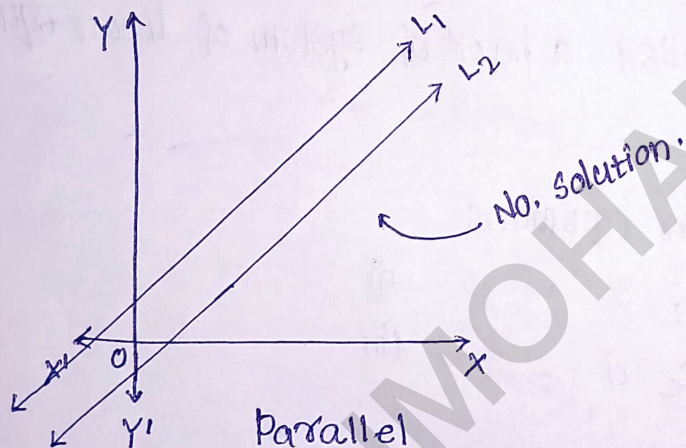
Graphical Presentation:



Intersecting



Coincident



Parallel

Solutions of pair of linear Equations:

Consider a pair of linear equations

$a_1x + b_1y + c_1 = 0$ ——— (i)

and $a_2x + b_2y + c_2 = 0$ ——— (ii)

I. Substitution Method:

From (i) $y = -\frac{a_1}{b_1}x - \frac{c_1}{b_1}$; if $b_1 \neq 0$

Substitute in equation (ii)

$$a_2x + b_2\left(-\frac{a_1}{b_1}x - \frac{c_1}{b_1}\right) + c_2 = 0$$

$$\Rightarrow \left(a_2 - \frac{b_2}{b_1}a_1\right)x + c_2 - \frac{b_2}{b_1}c_1 = 0$$

$$\Rightarrow \boxed{x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}}$$

From (ii), $\quad a_2\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}\right) + b_2y + c_2 = 0$

$$\Rightarrow b_2y = -c_2 - a_2\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}\right)$$

$$= \frac{-a_1b_2c_2 + a_2b_1c_2 - a_2b_1c_2 + a_2b_2c_1}{a_1b_2 - a_2b_1}$$

$$= \frac{b_2(c_1a_2 - c_2a_1)}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \boxed{y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}}$$

II. Elimination Method:

Eq^n(i) $\times b_2$: $\quad a_1b_2x + b_1b_2y + b_2c_1 = 0$

Eq^n(ii) $\times b_1$: $\quad a_2b_1x + b_1b_2y + b_1c_2 = 0$

$\quad (-) \qquad (-) \qquad (-) \qquad (-)$

$$\Rightarrow (a_1b_2 - a_2b_1)x = b_1c_2 - b_2c_1$$

$$\Rightarrow \boxed{x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}}$$

Substitute in equation(i)

$$a_1\left(\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}\right)+b_1y+c_1=0$$

$$\Rightarrow b_1y=-c_1-a_1\left(\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}\right)$$

$$=\frac{-a_1b_2c_1+a_2b_1c_1-a_1b_1c_2+a_1b_2c_1}{a_1b_2-a_2b_1}$$

$$=b_1\left(\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}\right)$$

$$\Rightarrow \boxed{y=\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}}$$

N.B. For 2 system of equation of the type

$$\frac{a_1}{x}+\frac{b_1}{y}=c_1 \quad \text{———(i)}$$

and $$\frac{a_2}{x}+\frac{b_2}{y}=c_2 \quad \text{———(ii)}$$

Take $\frac{1}{x}=X$ and $\frac{1}{y}=Y$ where $X\neq 0$ and $Y\neq 0$.

then Solve the above.

Ex: Solve $$\frac{5}{x+y}-\frac{2}{x-y}=-1 \quad \text{———(i)}$$

and $$\frac{15}{x+y}+\frac{7}{x-y}=10 \quad \text{———(ii)}$$

where $x\neq 0$ and $y\neq 0$.

Take $\frac{1}{x+y} = X$ ~~and~~ $\frac{1}{x-y} = Y$

From (i) and (ii)

$$5X - 2Y = -1 \quad \text{——— (iii)}$$

$$\text{and} \quad 15X + 7Y = 10 \quad \text{——— (iv)}$$

Eq^n (iii) $\times 7$: $35X - 14Y = -7$

Eq^n (iv) $\times 2$: $30X + 14Y = 20$

$\Rightarrow 65X = 13$

$\Rightarrow \boxed{X = \frac{1}{5}}$

From (iii) $2Y = 5X + 1$

$\Rightarrow Y = \frac{1}{2}\left(5\left(\frac{1}{5}\right)+1\right)$

$\Rightarrow \boxed{Y = 1}$

So $x + y = 5$ ——— (v)

and $x - y = 1$ ——— (vi)

$+$

$2x = 6$

$\Rightarrow \boxed{x = 3}$

From (vi) $y = x - 1 = 3 - 1$

$\Rightarrow \boxed{y = 2}$

Application to word problem

Ex: 5 pen and 6 pencils together cost Rs. 9 and 3 pens and 2 penciles cost Rs. 5. Find the cost of 1 pen and 1 pencil.

Solution

Let Cost of 1 pen = ₹ x

Cost of 1 pencil = ₹ y

ATQ, $5x + 6y = 9$ ——— (i)

and $3x + 2y = 5$ ——— (ii)

$$\text{Eq}^n\text{(i)} \times 1: \quad 5x + 6y = 9$$

$$\text{Eq}^n\text{(ii)} \times 3: \quad \underset{(-)}{9x} \underset{(-)}{+ 6y} = \underset{(-)}{15}$$

$$\cancel{14x} \; -4x = -6$$

$$\Rightarrow \boxed{x = \frac{3}{2}}$$

From (ii) $2y = 5 - 3x$

$$\Rightarrow y = \frac{1}{2}\left(5 - 3\left(\frac{3}{2}\right)\right)$$

$$\Rightarrow \boxed{y = \frac{1}{4}}$$

Hence cost of 1 pen ₹ $\frac{3}{2}$ and 1 pencil is ₹ $\frac{1}{4}$.

N.B. Upstream and Downstream:

Let speed of boat in ~~upstream~~ still water = x,

Speed of stream = y

then Speed ~~of~~ upstream (S_u) = $x - y$

Speed in downstream (S_d) = $x + y$

4. QUADRATIC EQUATION

A polynomial equation of the form "$ax^2+bx+c=0$", Where a,b,c are real number and $a \neq 0$ is called a quadratic equation.

Ex:

$$x^2+2x+1=0$$

Solution of a quadratic equation:

(I) Method of factorization:

~~coe~~ have to search ~~two~~ real number k_1 and k_2

Such that $ax^2+bx+c = ax^2+(k_1+k_2)x+\cancel{k_1k_2}\,c$

where $k_1+k_2 = b$ —— (i)

and $k_1k_2 = ac$ —— (ii)

$\therefore (k_1-k_2)^2 = (k_1+k_2)^2 - 4k_1k_2$

$= b^2-4ac$

$\Rightarrow k_1-k_2 = \cancel{b}\pm\sqrt{D}$; where D= discriminant —— (iii)

From (ii) and (iii)

$$\boxed{k_1 = \frac{b\pm\sqrt{D}}{2}} \text{ and } \boxed{k_2 = \frac{b-\sqrt{D}}{2}}$$

Ex: $2x^2+x-1=0$

Let k_1 and k_2 are two such real numbers such that

$$2x^2 + x - 1 = 2x^2 + (K_1 + K_2)x \cancel{+ K_1 K_2} - 1$$

Where $K_1 + K_2 = 1$ ——— (i)

$K_1 K_2 = -2$ ——— (ii)

$$K_1 - K_2^2 = (K_1 + K_2)^2 - 4K_1K_2$$

$$= 1^2 - 4(-2) = 9$$

$\Rightarrow K_1 - K_2 = \pm 3$

$\Rightarrow K_1 - K_2 = 3$ ——— (iii)

From (i) and (iii)

$\boxed{K_1 = 2}$ and $\boxed{K_2 = -1}$

$\therefore \quad 2x^2 + x - 1 = 2x^2 + (2-1)x + 2(-1)$

$$= 2x^2 + 2x - x - 2$$

$$= 2x(x+1) - 1(x+1)$$

$$= (2x-1)(x+1)$$

So, $2x^2 + x - 1 = 0$

$\Rightarrow (2x-1)(x+1) = 0$

$\Rightarrow 2x - 1 = 0$ or $x + 1 = 0$

$\Rightarrow \boxed{x = 1/2}$ or $\boxed{x = -1}$

II: Quadratic root method (Shreedharacharya's formula)

Consider $ax^2 + bx + c = 0$ —— (i)

$\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$\Rightarrow x^2 + 2\cdot\frac{b}{2a}\cdot x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$

$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$

$\Rightarrow x + \frac{b}{2a} = \pm\frac{\sqrt{D}}{2a}$

$\Rightarrow x = \frac{-b \pm \sqrt{D}}{2a}$

$\therefore \boxed{x = \frac{-b + \sqrt{D}}{2a}}$ and $\boxed{x = \frac{-b - \sqrt{D}}{2a}}$

$\therefore$ ~~From (i)~~ a

N.B. $ax^2 + bx + c = a(x - \alpha)(x - \beta)$

where $\alpha = \frac{-b + \sqrt{D}}{2a}$ and $\beta = \frac{-b - \sqrt{D}}{2a}$

Ex: $2x^2 + x - 1 = 0$

Here $a = 2$, $b = 1$ and $c = -1$

By Shreedharacharya's formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)}$

$= \frac{-1 \pm 3}{4}$

$\Rightarrow \boxed{x = \frac{1}{2}}$ or $\boxed{x = -1}$

Nature of roots

Discriminant $D = b^2 - 4ac$

(i) If $D = 0$, then there are ~~only one~~ two equal zeroes (two equal roots)

(ii) If $D > 0$, then there are two real zeroes.

(iii) If $D < 0$, there is not real zeroes.

Application of quadratic equation

Ex: A two digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

Solution

Let the ten digit be x. Then

ATQ, unit digit is $\frac{18}{x}$.

Again $\left(10x + \frac{18}{x}\right) - \left(10 \times \frac{18}{x} + x\right) = 63$

$\Rightarrow 9x - 9 \times \frac{18}{x} = 63$

$\Rightarrow x - \frac{18}{x} = 7$

$\Rightarrow x^2 - 7x - 18 = 0$

$\Rightarrow x^2 - 9x + 2x - 18 = 0$

$\Rightarrow (x-9)(x+2) = 0$

$\Rightarrow \boxed{x = 9}$ (only possible value)

and unit place $= \frac{18}{x} = 2$

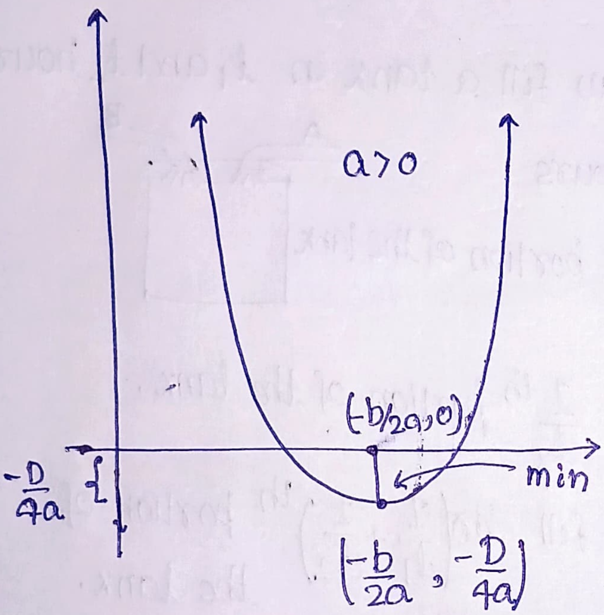
$\therefore$ The required number is 92.

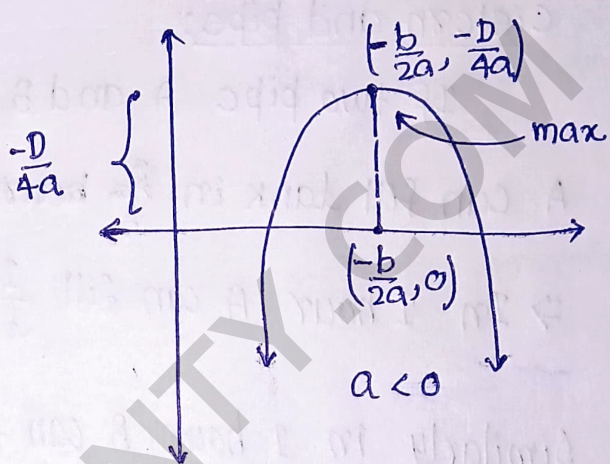
If $a > 0$ then minimum value exist.

Minimum value exist at $\frac{-b}{2a}$ and minimum value is $\frac{-D}{4a}$.

If $a < 0$, then maximum value exist.

Maximum value exist at $\frac{-b}{2a}$ and maximum value is $\frac{-D}{4a}$.





Ex   Find the maximum or minimum value of

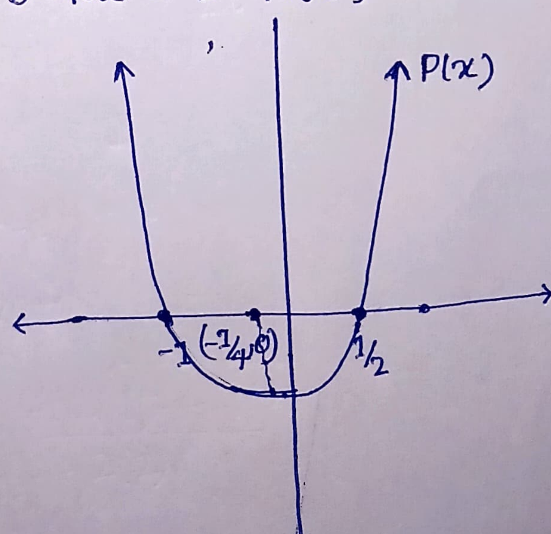
$P(x) = 2x^2 + x - 1$

As $a = 2 > 0$ and $D = b^2 - 4ac = 1^2 - 4(2)(-1) = 9 > 0$

$\therefore$ As $a > 0$; minimum value exist at $x = \frac{-b}{2a} = \frac{-1}{2(2)} = \frac{-1}{4}$

and minimum value is

$$-\frac{D}{4a} = \frac{-9}{4(2)} = \frac{-9}{8}$$



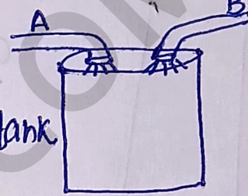
N.B (i) In Linear equation $\boxed{\text{Speed} = \frac{\text{distance}}{\text{time}}}$

$\Leftrightarrow$ $\boxed{\text{distance} = \text{speed} \times \text{time}}$

$\Leftrightarrow$ $\boxed{\text{time} = \frac{\text{distance}}{\text{speed}}}$

(ii) <u>Cistern and pipe:</u>

If two pipe A and B can fill a tank in t_1 and t_2 hours.



A can fill tank in t_1 ~~hours~~ hours

$\Rightarrow$ In 1 hour A can fill $\frac{1}{t_1}$ th portion of the tank.

Similarly in 1 hour B can fill $\frac{1}{t_2}$ th portion of the tank.

So, in 1 hour both A and B can fill ~~the~~ $\left(\frac{1}{t_1} + \frac{1}{t_2}\right)$ th portion of the tank.

So in $\frac{t_1 t_2}{t_1 + t_2}$ hours both A and B simultaneously can fill a tank.

5. ARITHMETIC PROGRESSIONS

Arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

The fixed number is called the common difference of the A.P.

N.B. (i) Common dibberence can be positive, negative or zero.

Exampe

The general form of an AP. is

$a, a+d, a+2d, a+3d, \ldots$

Here First term $= a$

Common difference $= d$

n^{th} term $T_n = a+(n-1)d$

The sum of first 'n' terms

$$S_n = a+(a+d)+(a+2d)+\cdots+a+(n-1)d$$

or

$$S_n = T_1 + T_2 + \cdots + T_n$$

$$\therefore \boxed{S_n = \frac{n}{2}\left[T_1 + T_n\right]}$$

OR

$$\Rightarrow \boxed{S_n = \frac{n}{2}\left[2a+(n-1)d\right]}$$

$$T_n = (T_1 + \cdots + T_n) - (T_1 + \cdots + T_{n-1})$$

$$\Rightarrow \boxed{T_n = S_n - S_{n-1}}$$

$$\Rightarrow \boxed{d = T_n - T_{n-1}}$$

N.B. (i) $d = T_n - T_{n-1} = (S_n - S_{n-1}) - (S_{n-1} - S_{n-2})$

$$\Rightarrow \boxed{d = S_n - 2S_{n-1} + S_{n-2}}$$

(ii) $T_n - T_{n-1} = T_{n-1} - T_{n-2} = a$

$$\Rightarrow \boxed{2T_{n-1} = T_n + T_{n-2}} \Leftrightarrow \boxed{T_{n-1} = \frac{T_n + T_{n-2}}{2}}$$

N.B. (i) If a, b and c are in AP then

$b - a = c - b$ then $\boxed{2b = a + c}$

(ii) $a-d, a, a+d$ are in AP.

(iii) $a-3d, a-d, a+d, a+3d$ are in AP.

(iv) $a-2d, a-d, a, a+d, a+2d$ are in AP.

(v) $a-4d, a-2d, a, a+2d, a+4d$, are in AP.

6. TRIANGLES

Triangle :

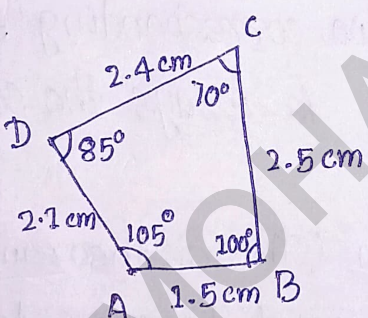
A polygon having 3 sides is called a triangle.

N.B. Two polygons having same number of sides are similar if

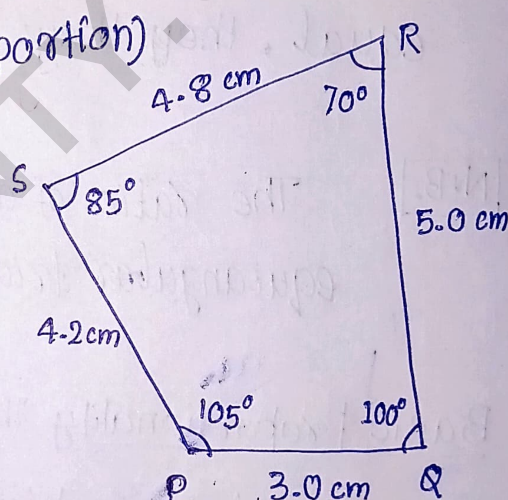
(i) their corresponding angles are equal

and (ii) their corresponding sides are in the same ratio (or proportion)

Example



(figure-1)



(figure-2)

Here figure-1 and figure-2 are both similar.

As (i) $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ and $\angle D = \angle S$

(ii) $\dfrac{PQ}{AB} = \dfrac{QR}{BC} = \dfrac{RS}{CD} = \dfrac{SP}{DA} = 2$

N.B. (i) All circles and squares are similar.

(ii) All equilateral triangles are similar.

Similarity of triangles:

Two triangles are similar, if

(i) their corresponding angles are equal and

(ii) their Corresponding sides are in the same ratio (or proportion).

Equiangular triangles:

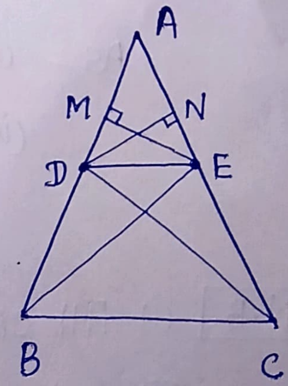
If corresponding angles of two triangles are equal, ~~they~~ they are known as equiangular triangles.

N.B. The ratio of any two corresponding sides in two equiangular triangles is always the same.

Basic Proportionality Theorem (Thales theorem)

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Proof: Given a triangle ABC.



D and E are two distinct points of AB and AE respectively such that $DE \parallel BC$.

Join BE and CD

Draw $EM \perp AB$ and $DN \perp AC$

$ar(\Delta ADE) = \frac{1}{2} \times AD \times EM$ ——— (i)

and also $ar(\Delta ADE) = \frac{1}{2} \times AE \times DN$

Again $ar(\Delta BED) = \frac{1}{2} \times DB \times EM$

and $ar(\Delta CDE) = \frac{1}{2} \times EC \times DN$

$\therefore$ Since $DE \parallel BC$ and ΔBED and ΔCDE both lies in between sharing same base DE.

So, $ar(\Delta BED) = ar(\Delta CDE)$

Now, $\frac{ar(\Delta ADE)}{ar(\Delta BED)} = \frac{ar(\Delta ADE)}{ar(\Delta CDE)}$

$\Rightarrow \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times DB \times EM} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN}$

$\Rightarrow \boxed{\frac{AD}{DB} = \frac{AE}{EC}}$

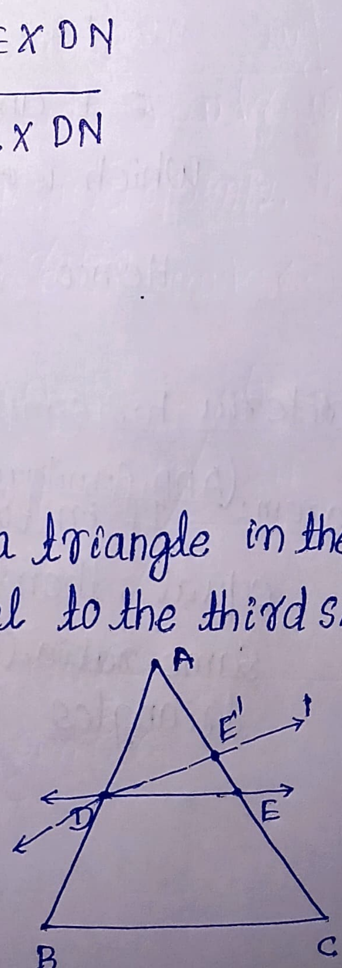
Converse of BPT

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Proof:

Given ~~DE || BC~~

$\frac{AD}{DB} = \frac{AE}{EC}$ ——— (i)



~~and assuming DE ∦ BC.~~
~~Assume~~
~~If~~ $DE \nparallel BC$, draw a line $DE' \parallel BC$.

So, by BPT,

$$\frac{AD}{DB} = \frac{AE'}{E'C} \quad \text{——— (ii)}$$

From (i) and (ii)

$$\frac{AE}{EC} = \frac{AE'}{E'C}$$

$$\Rightarrow \frac{AE}{EC} + 1 = \frac{AE'}{E'C} + 1$$

$$\Rightarrow \frac{AE + EC}{EC} = \frac{AE' + E'C}{E'C}$$

$$\Rightarrow \frac{AC}{EC} = \frac{AC}{E'C}$$

$$\Rightarrow EC = E'C$$

$\therefore$ E and E' are same or coincide

Which is contradiction to our assumption

Hence $DE \parallel BC$.

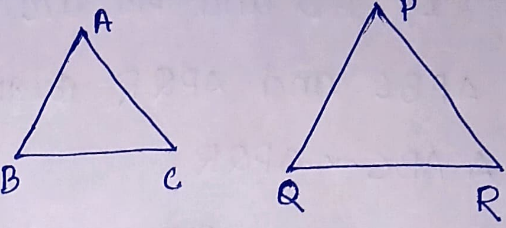
Criteria For similarity of Triangles:

I. Theorem: (AAA Similarity Criterion)
If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.

Explaination: If $\triangle ABC$ and $\triangle PQR$, $\angle A = \angle P$, $\angle B = \angle Q$, and $\angle C = \angle R$

then $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{CA}{RP}$

and so, $\triangle ABC \sim \triangle PQR$



$\boxed{\text{N.B.}}$ The above is also called AA-Similarity criterion.

II. SSS Similarity Criterion:

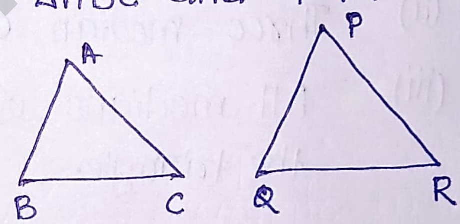
If in two triangles, sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence two triangles are similar.

Explanation: In two triangles $\triangle ABC$ and $\triangle PQR$

$\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR}$

then $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$

So, $\triangle ABC \sim \triangle PQR$



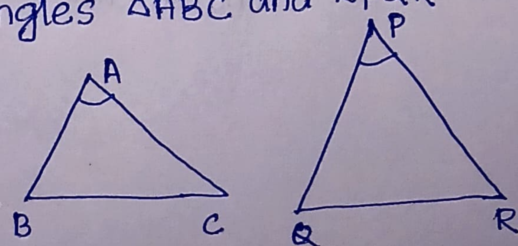
III. SAS Similarity criterion:

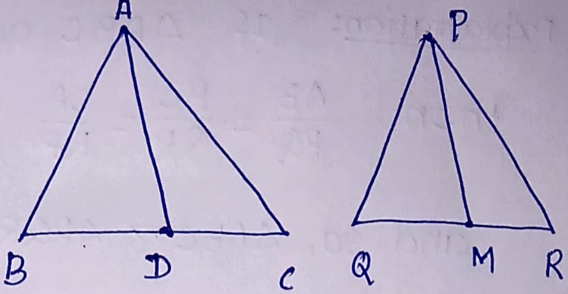
If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportionals then the two triangles are similar.

Explanation: In two triangles $\triangle ABC$ and $\triangle PQR$

If $\angle A = \angle P$ and

$\dfrac{AB}{PQ} = \dfrac{AC}{PR}$ then $\triangle ABC \sim \triangle PQR$.





N.B. If AD and PM are median of ΔABC and ΔPQR and $\Delta ABC \sim \Delta PQR$.

Then $\frac{AD}{PM} = \frac{BC}{QR}$

and $\Delta ABD \sim PQM$

Median:

A line segment joining vertex of triangle to it mid-point of opposite sides is called a median of that triangle.

N.B. (i) There are three median for any triangle.

(ii) Three median are concurrent.

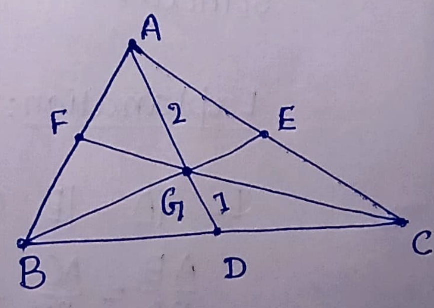
(iii) All medians of any type of triangle are lies inside the triangle.

Centroid:

The point of concurrent ~~(intersection)~~ of median is called centroid of triangle and it is denoted by G.

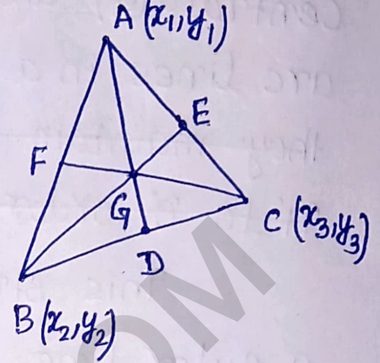
N.B. (i) Centroid of a triangle divide median in 2:1 ratio from vertex to the midpoint order.

(ii) Centroid of any type of triangle is always lies inside the triangle.



~~For~~ Theorem

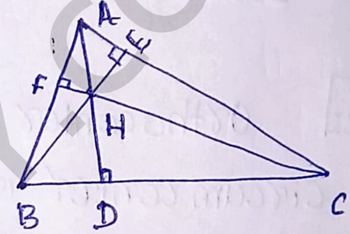
In a ΔABC, with vertices $A=(x_1,y_1)$, $B=(x_2,y_2)$ and $C=(x_3,y_3)$ then centroid of the triangle coordinate are $G=\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.



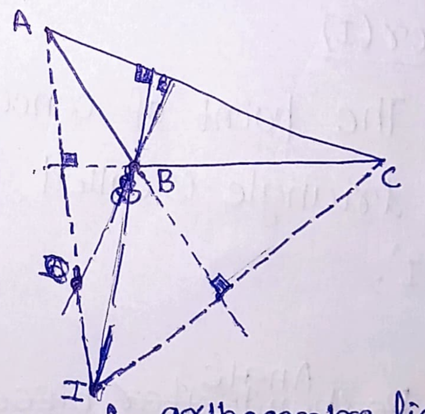
Ortho-centre (H)

The concurrent of altitudes of a triangle is called ~~the~~ an orthocentre of a triangle.

It is denoted by H.



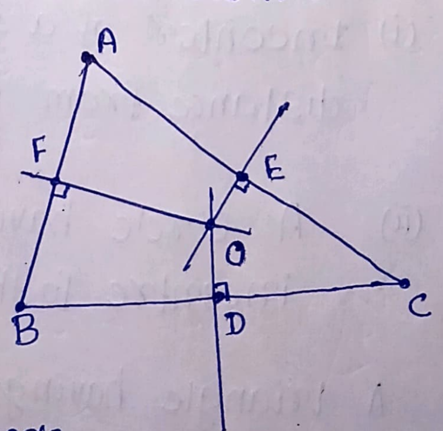
N.B. Orthocentre of a triangle may or may not be lies inside a triangle.



orthocentre lies outside

Circum center (O)

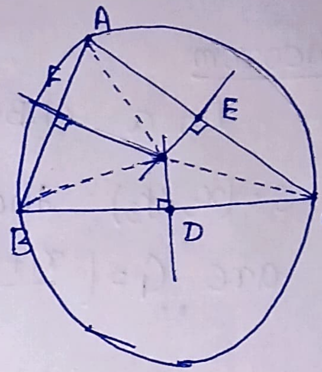
The concurrent point of perpendicular bisectors of a triangle is called ~~the~~ a circumcenter of a triangle.



N.B. (i) orthocentre are equal distance from all three vertices of triangle.

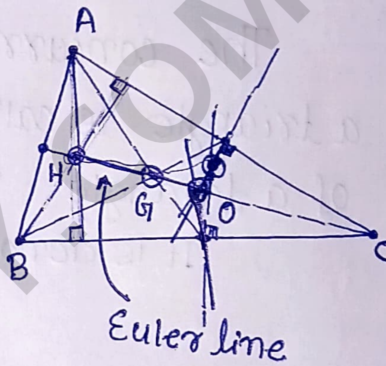
(ii) A circle having this equal length (radius) can be draw, where all three vertices lies on ~~the~~ circle.

Euler line:



- In any triangle orthocenter (H), Centroid (G) and circumcenter (O) are lines in a straight line and ~~they maintain 2:1 ratio from 'H' to 'O' order.~~

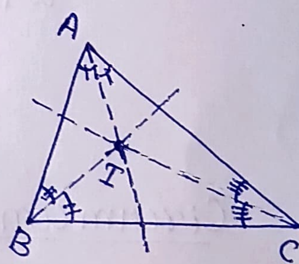
This straight line is called a Euler line.



N.B. Orthocenter, centroid and circum center in the euler line maintain 2:1 ratio.

Incenter (I)

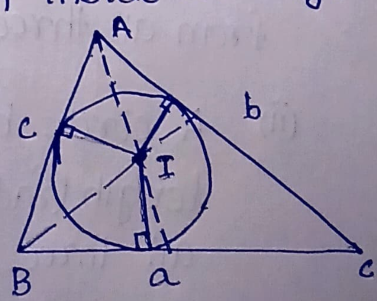
The point of concurrent of angle bisectors of a triangle is called incenter and is denoted by 'I'.



N.B. Angle ~~Perpendicular bisectors~~

(i) Incenter of a triangle are same perpendicular distance from all three sides.

(ii) A circle having this perpendicular length from incentre to the sides can be drawn inside a triangle.

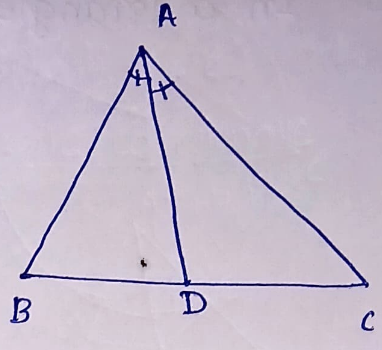


(iii) A triangle having vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ then its incenter coordinate is

$$I = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$$

Theorem:

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.



Explanation:

If AD is an internal bisector of $\triangle ABC$ ~~from~~ of $\angle A$, which ~~then~~ touch BC at D, then

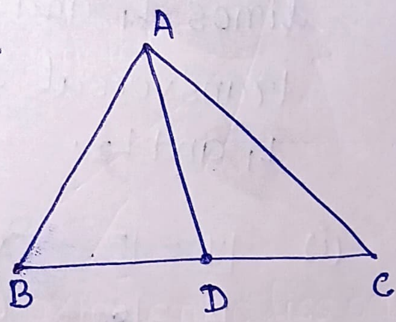
$$\frac{AB}{AC} = \frac{BD}{DC}$$

N.B. **Theorem:**

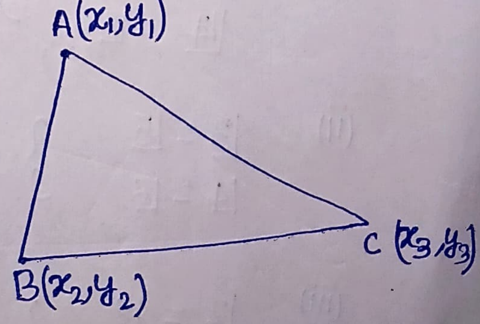
If a line through one vertex of a triangle divides the opposite sides in the ratio of other two sides, then the line bisects the angle at the vertex.

Explanation:

In a triangle ABC, if D is a point on BC such that $\frac{BD}{DC} = \frac{AB}{AC}$, then AD is an angle bisector of $\angle A$.



Area of a triangle:



$\boxed{I.}$ In a triangle ABC having vertices, $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ then

$ar(\triangle ABC)$ = non-negative value (absolute value of) $\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

II. In a triangle ABC, the $ar(\Delta ABC) = \frac{1}{2} \times AB \times AC \times \sin A$

or

$\frac{1}{2} \times BA \times BC \times \sin B$

or

$\frac{1}{2} \times CB \times CA \times \sin C$

Also $ar(\Delta ABC) = AB \times AC \times \sin\frac{A}{2} \times \cos\frac{A}{2}$

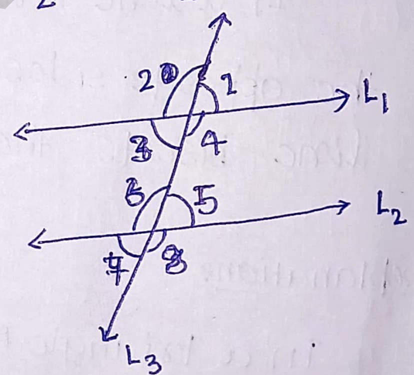
OR

$= BA \times BC \times \sin\frac{B}{2} \times \cos\frac{B}{2}$

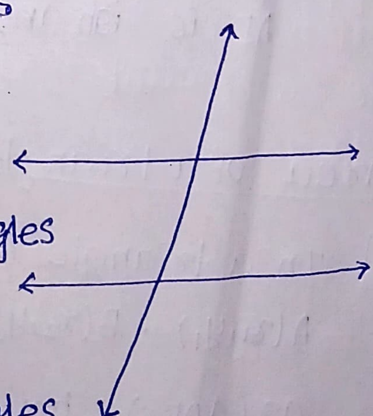
or

$= CB \times CA \times \sin\frac{C}{2} \times \sin\frac{C}{2}$

N.B. (i) Consider two parallel lines L_1 and L_2 and another transversal line L_3. intersect L_1 and L_2.



(i) $\angle 1 = \angle 5$
$\angle 2 = \angle 6$
$\angle 3 = \angle 7$
$\angle 4 = \angle 8$
} corresponding angles



(ii) $\angle 3 = \angle 5$
$\angle 4 = \angle 6$
} alternate interior angles

(iii) $\angle 2 = \angle 8$
$\angle 1 = \angle 7$
} alternate exterior angles

(iv) $\angle 1 = \angle 3$, $\angle 4 = \angle 2$ vertical angles

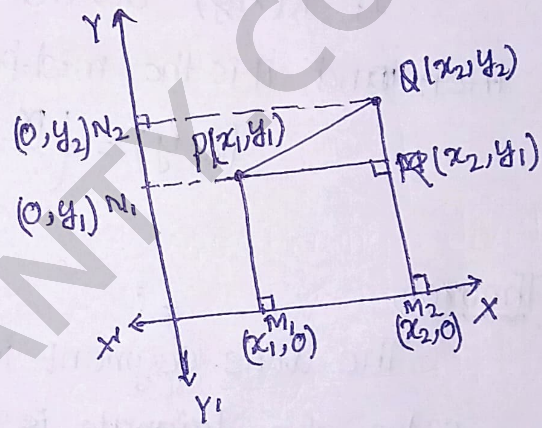
(v) $\angle 4 + \angle 5 = 180$ and $\angle 4$ & $\angle 5$ are called consecutive interior angles.

7. COORDINATE GEOMETRY

To locate position of any point in plane is represented by a pair of coordinate axis. x-coordinat (abscissa) and y-coordinate (ordinate).

Distance Formula:

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points in xy plane.



By pythagorean theorem

$$PQ^2 = PR^2 + QR^2$$

$$= M_1M_2^2 + N_1N_2^2$$

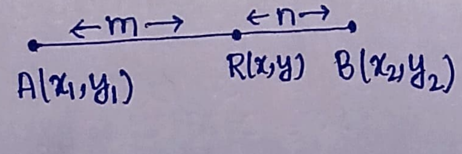
$$= (x_2-x_1)^2 + (y_2-y_1)^2$$

$$\Rightarrow \boxed{PQ = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}}$$

Section formula:

The coordinates of the point $P(x, y)$ which divides the line segment joining the point $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m:n$ are

$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$



N.B. take $\frac{m}{n} = k$ then in Section formula $R(x,y)$ divide AB internally in $k:1$ ratio. and coordinate becomes

$$R(x,y) = \left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1} \right)$$

mid-point formula:

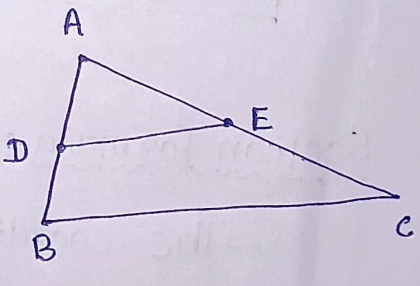
If $R(x,y)$ divide the line joining $A(x_1,y_1)$ and $B(x_2,y_2)$ ~~then~~ and it is the mid-point then

$$(x, y) = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$$

Theorem

The line segment joining the mid-points of two sides of a triangle is parallel to the third side and half of its.

Explanation:



In a triangle ABC, say D and E are mid-points of AB and AC respectively.

Then $DE \parallel BC$ and

$$DE = \frac{1}{2} BC.$$

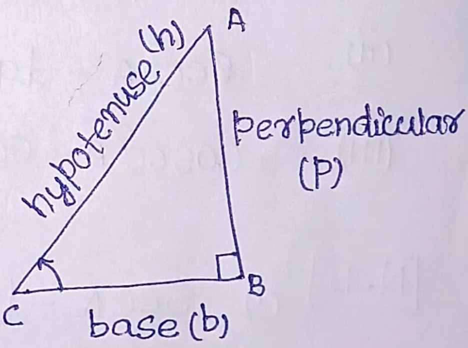
8. INTRODUCTION TO TRIGONOMETRY

The word "trigonometry" is derived from the Greek words "tri" (meaning three), 'gon' (meaning sides) and 'metron' (meaning measure).

Trigonometry is the study of relationships between the sides and angles of a triangles.

Trigonometric Ratios:

Consider a right angle triangle.



Here $\angle ACB$ is an acute angle.

The "side opposite to angle $\angle C$" is called perpendicular.

The "side adjacent to angle $\angle C$" is called base.

The "side opposite of $90°$" is called hypotenuse.

Define Sine of $\angle C = \frac{AB}{AC} \Rightarrow \boxed{\sin C = \frac{p}{h}}$

Cosine of $\angle C = \frac{BC}{AC} \Rightarrow \boxed{\cos C = \frac{b}{h}}$

tangent of $\angle C = \frac{AB}{BC} \Rightarrow \boxed{\tan C = \frac{p}{b}}$

Cosecant of $\angle C = \frac{AC}{AB} \Rightarrow \boxed{\text{cosec } C = \frac{h}{p}}$

Secant of $\angle C = \frac{AC}{BC} \Rightarrow \boxed{\sec C = \frac{h}{b}}$

Cotangent of $\angle C = \frac{BC}{AB} \Rightarrow \boxed{\cot C = \frac{b}{p}}$

Angle	0°	30°	45°	60°	90°	
Sin	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	
Cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	undefined	

Trigonometry Identity:

(i) $\sin^2 A + \cos^2 A = 1$; f

(ii) $\sec^2 A - \tan^2 A = 1$; for $0° \leq A < 90°$

(iii) $\text{cosec}^2 A - \cot^2 A = 1$; for $0° < A \leq 90°$

$\boxed{\text{N.B}}$ (i) $\sec A - \tan A = \frac{1}{\sec A + \tan A}$

(ii) $\text{cosec} A - \cot A = \frac{1}{\text{cosec} A + \cot A}$

(iii) $\tan A = \frac{\sin A}{\cos A}$; $\cot A = \frac{\cos A}{\sin A}$; $\text{cosec} A = \frac{1}{\sin A}$; $\sec A = \frac{1}{\cos A}$

Example

PT $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$

$$= \frac{\dfrac{\sin\theta - \cos\theta + 1}{\cos\theta}}{\dfrac{\sin\theta + \cos\theta - 1}{\cos\theta}} = \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

$$= \frac{(\tan\theta + \sec\theta) - 1}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\dfrac{1}{\sec\theta - \tan\theta} - 1}{\tan\theta - \sec\theta + 1} \quad \left(\because \sec\theta + \tan\theta = \frac{1}{\tan\theta \sec\ldots}\right)$$

$$= \frac{1 - \sec\theta + \tan\theta}{1 - \sec\theta + \tan\theta} \times \frac{1}{\sec\theta - \tan\theta}$$

$$= 1 \times \frac{1}{\sec\theta - \tan\theta} = \frac{1}{\sec\theta - \tan\theta}$$

N.B.

$\tan 15^\circ = 2 - \sqrt{3}$	$\sin 75^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$	$\cos 15^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$
$\tan 75^\circ = 2 + \sqrt{3}$	$\sin 54^\circ = \dfrac{\sqrt{5}+1}{4}$	$\cos 75^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$
$\sin 15^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$		
$\sin 18^\circ = \dfrac{\sqrt{5}-1}{4}$		

N.B.

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$.

$\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$

$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

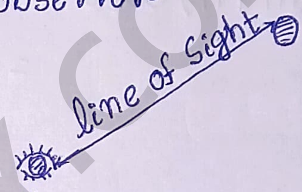
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$$

Q

9. SOME APPLICATIONS TRIGONOMETRY

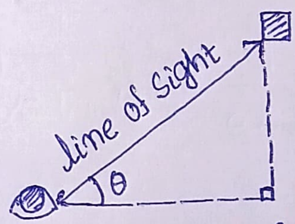
Line of Sight:

It is the line drawn from the eye of an observer to the point in the object viewed by the observer.



Angle of elevation:

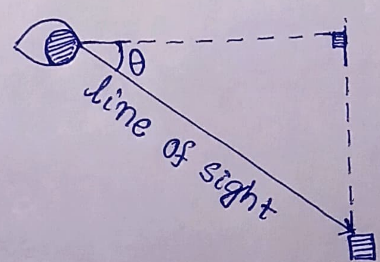
It is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level.



angle of elevation

Angle of depression:

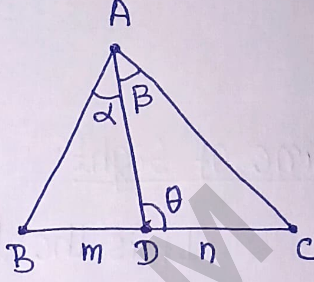
It is the angle formed by the line of sight with the horizontal when the points is below the horizontal level.



angle of depression

m-n theorem:

In a ΔABC, D is a point on the line BC such that $BD : DC = m : n$ and $\angle ADC = \theta$, $\angle BAD = \alpha$, $\angle DAC = \beta$, then



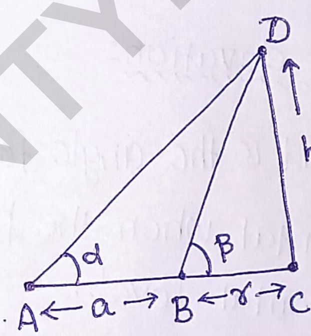
$$(m+n)\cot\theta = m\cos\alpha - n\cot\beta$$

$$= n\cot B - m\cos C$$

$\boxed{\text{N.B}}$

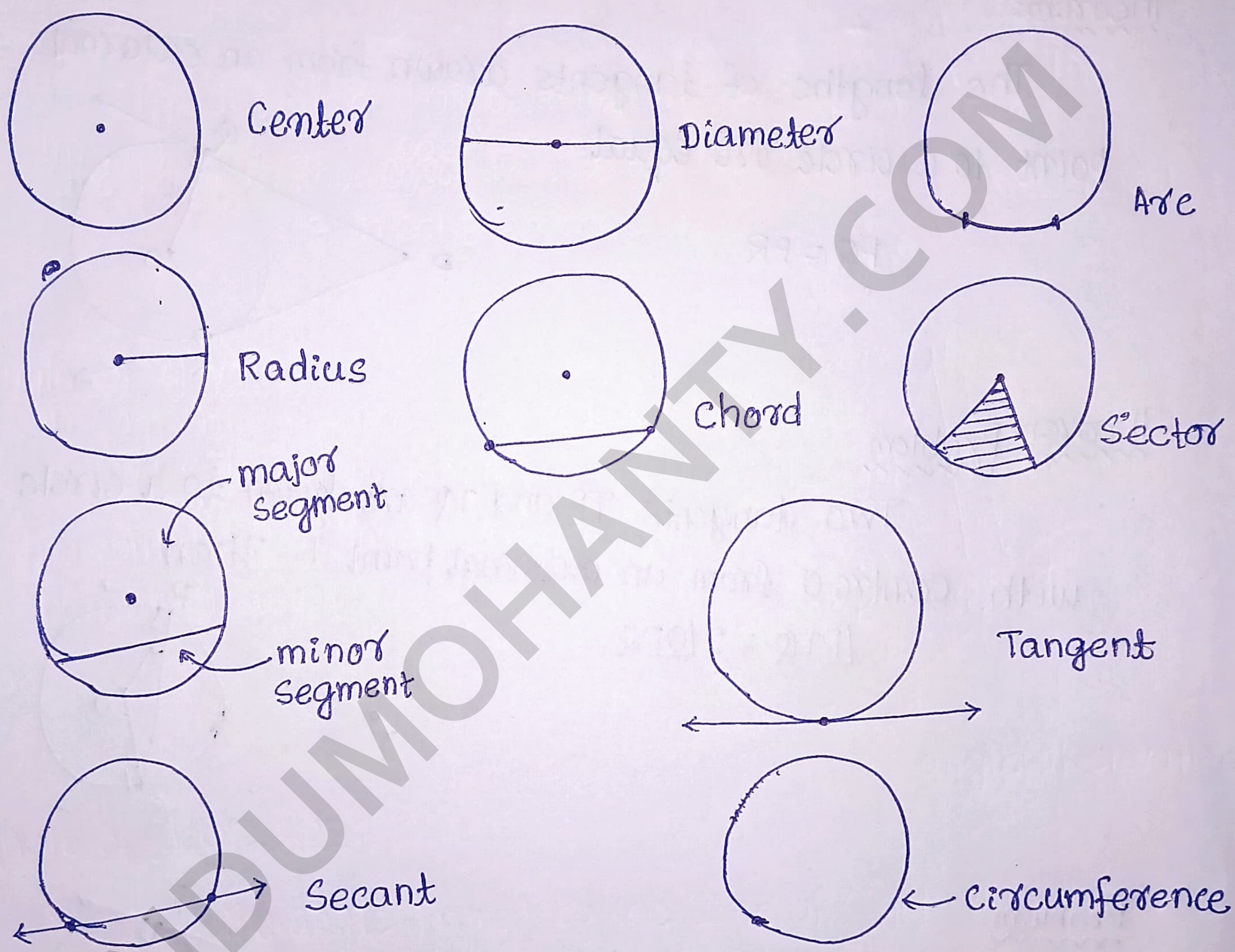
(i) In this figure

$$a = h(\cot\alpha - \cot\beta)$$



10. CIRCLES

A circle is a collection of all points in a plane which are at a constant distance (radius) from a fixed point (centre).

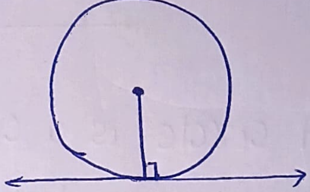


Tangent: ~~A circle~~ A)
A line that intersects the circle at only one point.

Secant:
A line which intersects the circle at two points.

Chord: A line segment whose ends points are lies on a circle.

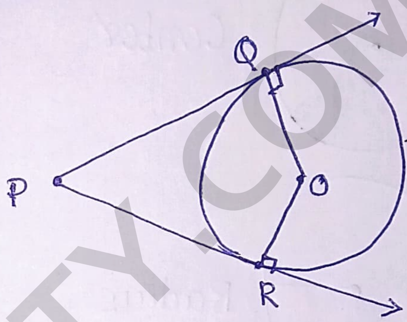
Theorem: The tangent at any point of a circle is perpendicular to the radius through the point of contact.



Theorem:

The lengths of tangents drawn from an external point to a circle are equal.

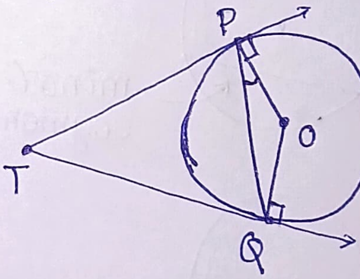
$PQ = PR$



~~Theorem~~ **Problem**

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Then

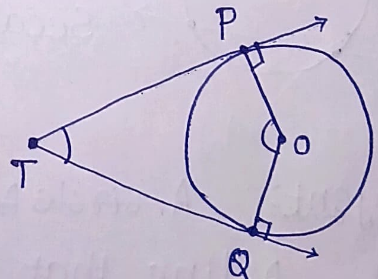
$\angle PTQ = 2\angle OPQ$



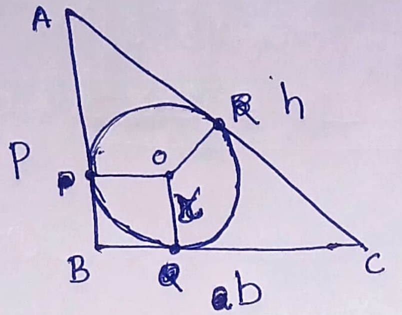
Problem

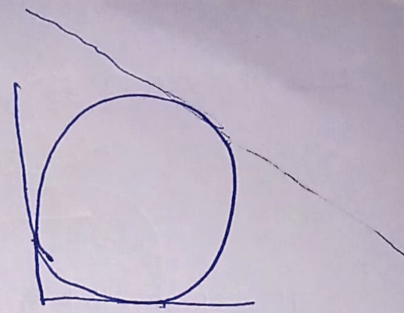
Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Then

$\angle POQ + \angle PTQ = 180^\circ$



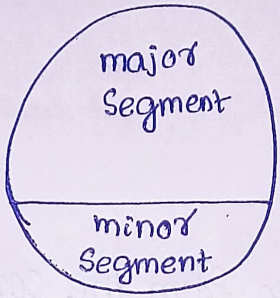
N.B.





$$\boxed{r = \frac{a+b-c}{2}} \quad \text{or} \quad \boxed{r = \frac{p+b-h}{2}}$$

11. AREA RELATED TO CIRCLES



$\boxed{\text{N.B.}}$ (i) Area of Sector having angle θ between two radii

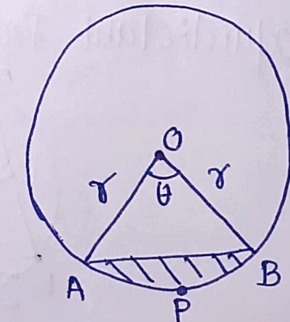
$$ar(\text{Sector}) = \frac{\theta}{360^\circ} \times \pi r^2$$

(ii) perimeter of a sector having θ between two radii

$$ar(\text{Sector}) = \frac{\theta^\circ}{360^\circ} \times 2\pi r$$

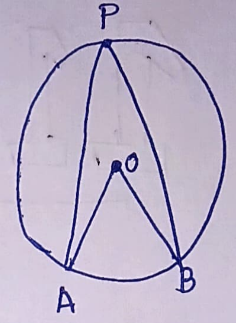
$\boxed{\text{N.B.}}$ $ar(\text{APB Segment}) = ar(\text{AOB Sector}) - ar(\triangle OAB)$

$$= \frac{\theta^\circ}{360^\circ} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$$



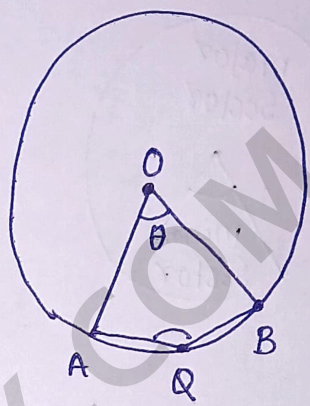
$\boxed{\text{N.B.}}$ $$ar(\triangle OAB) = \begin{cases} \frac{1}{2} r^2 \sin\theta \; ; \; \theta = \text{acute} \\ r^2 \sin\left(\frac{\theta}{2}\right) \cdot \cos\left(\frac{\theta}{2}\right) \; ; \; \theta = \text{obtuse} \end{cases}$$

N.B. $\angle AOB = 2\angle APB$

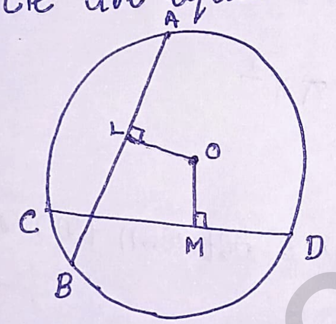


(i) The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

(ii) $\angle AQB = 180^\circ - \frac{\theta}{2}$



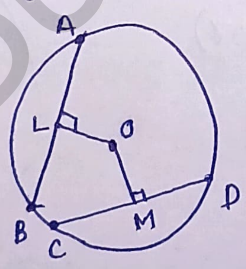
(iii) Chords equidistance from the centre of a circle are equal in length.



$OL = OM$

then $AB = CD$

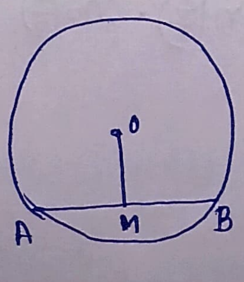
(iv) Equal chords of a circle (or of congruent circles) are equidistant from the centre.



$AB = CD$

then $OL = OM$.

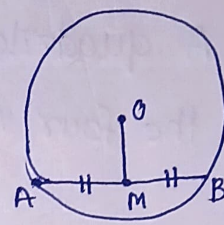
(v) The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.



If $OM \perp AB$

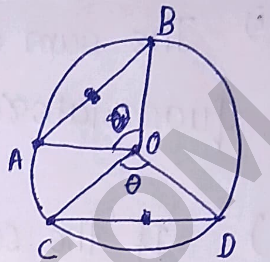
then $AM = BM$

(vi) The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.



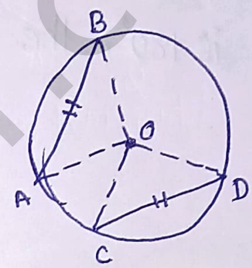
(vi) If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.

If $\angle AOB = \angle COD$ then $AB = CD$



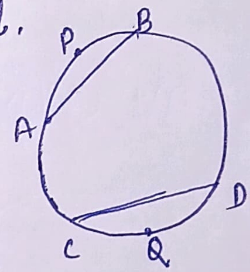
(vii) Equal chords of a circle subtend equal angles at the centre.

If $AB = CD$ then $\angle AOB = \angle COD$



N.B.- (i) If two chords of a circle are equal, then their corresponding arcs are congruent and conversely, if two arcs are congruent, then their corresponding chords are equal.

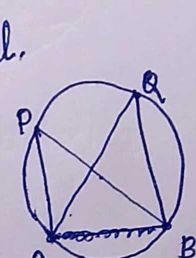
If $AB = CD$ then $\overset{\frown}{APB} = \overset{\frown}{CQD}$



(ii) Congruent arcs (or equal arcs) of a circle subtend equal angles at the centre.

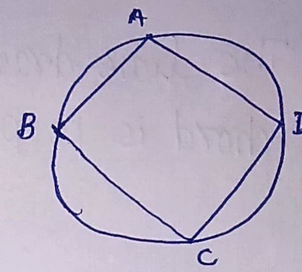
(iii) Angles in the same segment of a circle are equal.

$\angle APB = \angle AQB$, where P & Q are lies on the same ~~seg~~ arc of a segment.



(iv) If a line segment joining two points ~~two~~ subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e., they are concylic).

Cyclic Quadrilaterals:



A quadrilateral is called a cyclic if all the four vertices of it lie on a circle.

N.B. (i) The sum of either pair of opposite angles of a cyclic quadrilateral is 180°

(ii) If the sum of a pair of opposite angles of a quadrilateral is 180°, the quadrilateral is cyclic.

12. Surface Areas And Volumes:

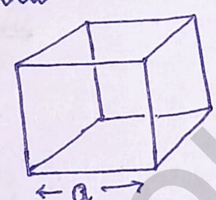
Volume: The space contained by an object or a ~~3D~~ shape is called its volume. ~~of it~~

Total Surface Area (TSA)

The total surface area of a solid is the sum of the curved surface area of each of the individual parts.

Types of Solids:

(i) **Cube:**



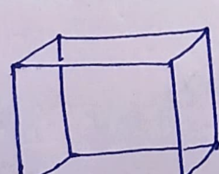
A cube is a three-dimensional solid object bounded by six equal faces, vertices, and edges.
A regular hexahedron is called a cube.

- If edge of a cube = a unit, then

Volume = a^3 units cube or a^3 units^3

TSA = $6a^2$ square units or $6a^2$ unit^2.

(ii) **Cuboid:**

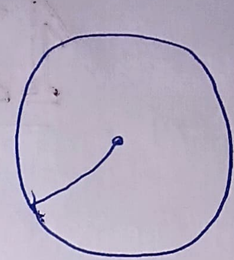


If length = l unit
breadth = b unit
height = h unit

Volume = $l \times b \times h$ unit^3.

TSA = $2(l \times b + b \times h + h \times b)$ unit^2.

(iii) **Sphere:**



Sphere

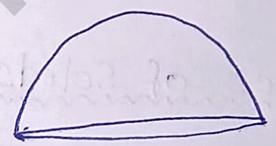
A shape having a fixed ~~center~~ point and a fixed distance in space is called a sphere.

Fixed point is called center and fixed distance is called radius.

Volume $= \frac{4}{3}\pi r^3$

Surface area $= 4\pi r^2$

(iv) **Hemisphere:**



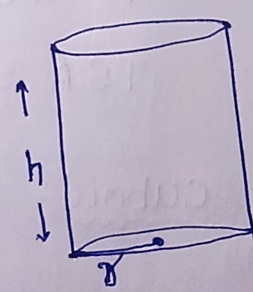
Half of a sphere is called a hemisphere.

Volume $= \frac{2}{3}\pi r^3$

TSA = Curved surface area + base area

$= 2\pi r^2 + \pi r^2 = 3\pi r^2$

(v) **Cylinder:** (Right circular cylinder)



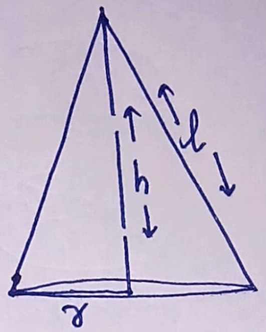
Volume $= \pi r^2 h$

TSA = CSA + 2 Base (Disk)

$= 2\pi r h + 2 \times \pi r^2$

$= 2\pi r (h + r)$

(vi) Cone: (Right circular cone)



Volume $= \frac{1}{3}\pi r^2 h$

~~cs~~ CSA $= \pi r l$

TSA $= \pi r l + \pi r^2$

Where ~~l~~ $l = \sqrt{h^2 + r^2}$

13. STATISTICS

Measures of central tendency

The statistical techniques used to find out the centre of distribution ~~rather than~~ are referred as measures of Central tendency.

The commonly used measures of central tendency (or averages) are:

- Mean
  - Arithmetic Mean ✓
  - Geometric Mean
  - Harmonic mean
  - weighted mean
- Median ✓
- Mode ✓

Data

- ~~Raw Data~~ Grouped frequency Distribution
  - Inclusive
  - Exclusive
- Ungrouped frequency distribution
  - Raw Data
  - Simple frequency distribution

(I) Arithmetic mean (Mean) The average of a data is called it AM.

(i) Direct method:

$$\bar{x} = \begin{cases} \dfrac{\sum x_i}{n} & \text{(Raw Data)} \\[2ex] \dfrac{\sum f_i x_i}{\sum f_i} & \text{(simple frequence distribution)} \\[2ex] \dfrac{\sum f_i m_i}{\sum f_i} & \text{(grouped frequency distribution)} \end{cases}$$

(ii) Assumed mean method:

$$\bar{x} = \begin{cases} a + \dfrac{\sum d_i}{n} & \text{(Raw data)}\quad d_i = x_i - a \\[2ex] a + \dfrac{\sum f_i d_i}{\sum f_i} & \text{(Simple frequency distribution)}\quad d_i = x_i - a \\[2ex] a + \dfrac{\sum f_i d_i}{\sum f_i} & \text{(grouped frequency distribution)} \\ \text{where } d_i = m_i - a & \end{cases}$$

(iii) Step-deviation method:

$$\bar{x} = \begin{cases} a + \dfrac{\sum u_i}{n} \times h \;;\; h \neq 0 & \text{(Raw Data)} \\[2ex] a + \dfrac{\sum f_i u_i}{\sum f_i} \times h \;;\; h \neq 0 & \text{(Simple frequency Distribution)} \\[2ex] a + \dfrac{\sum f_i u_i}{\sum f_i} \times h \;;\; h \neq 0 & \text{Grouped} \\ \text{generally } h = \text{length of interval} & \end{cases}$$

where $u_i = \dfrac{x_i - a}{h}$

N.B. If in a grouped frequency distribution ~~then convert to~~ if data in ~~exclusive~~ inclusive form the convert to exclusive form then apply methods.

(II) Mode: It is value among the observations having maximum frequency (which occur most often).

$$\text{median} = \begin{cases} \text{which occur most often} \;;\; \text{for Raw data} \\ \text{Which having maximum frequency} \;;\; \text{for Simple frequency} \\ l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h \;;\; \text{grouped frequency distribution.} \end{cases}$$

Where l = lower limit of the modal class

f_1 = frequency of the modal class

f_0 = Preceding frequency of the modal class

f_2 = Succeeding frequency of the modal class

h = length of class interval ~~of modal class~~.

$\boxed{\text{N.B.}}$ In grouped frequency data

$$m_i = x_i = \text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class Limit}}{2}$$

III. Median ~~of grouped data:~~

The middle-most observation of the data is called it's median.

$$\text{Median (M)} = \begin{cases} \left(\frac{n+1}{2}\right)\text{th observation; if } n \text{ is odd} \\ \dfrac{\left(\frac{n}{2}\right)\text{th} + \left(\frac{n}{2}+1\right)\text{th}}{2} \text{ observation; if } n \text{ is odd} \\ l + \left(\dfrac{\frac{n}{2} - C_f}{f}\right) \times h \text{ ; for grouped frequency data.} \end{cases}$$

} for Raw and Simple frequency Data

Where l = lower limit of median class

n = number of observations

C_f = cumulative frequence of class preceding the median class

f = frequency of median class

h = class size (assuming class size to be equal).

$\boxed{\text{N.B.}}$ (i) $\bar{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow \bar{x}(\Sigma f_i) = \Sigma(f_i x_i) \Rightarrow \bar{x}N = \Sigma f_i x_i$

(ii) $\bar{x} = \dfrac{\sum x_i}{n} \Rightarrow \boxed{n\bar{x} = \sum x_i}$

14. PROBABILITY

In the theory of probability it deal with events which are outcomes of an experiment. The word "experiment" means an operation which can produce some well-defined outcomes(s).

There are two types of experiments

× (i) Deterministic

✓ (ii) Random or Probabilistic

Elementary Event:

An outcome of an random experiment is called an elementary event.

Compound event:

An event associated to a random experiment is said to occur if any one of the elementary events associated to the event A is an outcome.

Negation of an event:

Define an event "not A" which occurs when and only when A doesn't occur. is denoted by A^c or A' or $\bar{A}$ and called negation of event A.

$\boxed{\text{N.B.}}$ Event 'A' occurs iff $\bar{A}$ doesn't occur.

Probability of an event

If E is an event of a random experiment then empirical probability of an event E is denoted and defined as

$$P(E) = \frac{\text{Number of trials in which the event happens}}{\text{Total number of trials}}$$

The theoretical probability of an event E, written and defined as $P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$

$$= \frac{n(E)}{n(S)}$$

Sample space (S)

It is the collection of all outcomes of an experiment. It is denoted by S.

N.B. (i) If $P(E) = 0$ then E is called a impossible event

If $P(E) = 1$ then E is called a sure event or certain event.

(ii) $P(E) + P(\bar{E}) = 1$

where E and $\bar{E}$ are complement of each other.

(iii) $E \cap \bar{E} = \phi$ and $E \cup \bar{E} = S$.

(iv) For any event E, $0 \leq P(E) \leq 1$.

I. COIN

A coin has two sides Head (H) and Tail (T).

So getting head and tail both has same changes in each trail. i.e., $1/2$.

(i) Sample space in 1 trail is

$S = \{H, T\} \Rightarrow n(S) = 2^1$

(ii) Sample space in 2 trails are

$S = \{HH, HT, TH, TT\} \Rightarrow n(S) = 2^2$

(iii) Sample space in 3 trails are

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$\Rightarrow n(S) = 2^3 = 8$

$\boxed{N.B.}$ (i) In n-trials no. of elementary events in the sample space is $n(S) = 2^n$.

(ii)

II. DICE

A dice has 6 sides as 1, 2, 3, 4, 5 and 6.

Each elementary events are equiprobable.

(i) Sample space of 1 dice draw is

$S = \{1, 2, 3, 4, 5\}$

$\Rightarrow n(S) = 6$

(ii) Sample space of 2 dice draw is

$S = \{(1,1), (1,2), \ldots, (6,6)\}$

$n(S) = 6 \times 6 = 36$

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Diagonal elements (DE):

= (1,1), (2,2), (3,3), (4,4), (5,5), (6,6)

Others are Non-diagonal (NDE)

(ii) Sample space 3 dice draw

$S = \{(1,1,1), \ldots, (6,6,6)\}$

$n(S) = 6 \times 6 \times 6 = 6^3 = 216$

$\boxed{\text{N.B.}}$ (i) In two dice drawn

Sum of two dice can be 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.

		No of events
Sum of 2 :	(1,1)	1
Sum of 3 :	(1,2), ~~(2,2)~~, (2,1)	2
Sum of 4 :	(1,3), (2,2), (3,1)	3
Sum of 5 :	(1,4), (2,3), (3,2), (4,1)	4
Sum of 6 :	(1,5), (2,4), (3,3), (4,2), (5,1)	5
Sum of 7 :	(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)	6
Sum of 8 :	(2,6), (3,5), (4,4), (5,3), (6,2)	5
Sum of 9 :	(3,6), (4,5), (5,4), (6,3)	4
Sum of 10 :	(4,6), (5,5), (6,4)	3
Sum of 11 :	(5,6), (6,5)	2
Sum of 12 :	(6,6)	1

$\boxed{\text{OR}}$

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

E = Sum of 2 dice	2	3	4	5	6	7	8	9	10	11	12
P(E)	2/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

(iii) product of two dice: ~~1, 2, . . . , 36~~ ~~1, 2, 3, 4, 5, 6, 8, 10, 12,~~

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

product: 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30 and 36.

III : CARD

There are 52 different cards. and 4 types of cards are Spade(♤), Club(♧), Diamond(◇) and Heart(♡).

Each of type are 13 no. of cards and they are 2, 3, 4, 5, 6, 7, 8, 9, J, Q, K and A.

There are two colours cards they are Red and Black. where 26 (Diamong & Heart) are red and 26 (Space and club) are black.

J, Q, K are called face cards.

There are 3X4 = 12 face cards.

A is called an ace card.

There are 4 ace card.

IV. YEAR

In every 4 yeare there is a leap year except century year. and other 3 are ~~Im~~ non-leap year.

Leap-year is divisible by 4.

Every 4th century is a leap year.

Ordinary / non-Leap year:

A year having 365 days, is called ordinary or non-leap year. Here 52 week + 1 day.

Leap year:

A year having 366 days is called a leap year. Here 52 week + 2 days

Odd Days: Number of days more than the complete number of weeks in a given period is called odd days.

~~Im~~ Non-leap year	Leap-year
1 odd day can be	2 odd day can be
Monday	Monday, Tuesday
Tuesday	Tuesday, wednesday
wednesday	wednesday, Thursday
Thursday	Thursday, Friday
Friday	Friday, saturday
Saturday	saturday, sunday
sunday	sunday, Monday