Half Yearly-2025
Class-XI (Science)
MATH-041

1. (d) NOT

2. (c) $\{x \in \mathbb{R} : 2x^2 - 1 = 0\}$

3. (c) 6, 3

4. (c) $\mathbb{R}^-$

5. (c) and (d)

6. (b) 1

7. (d) $2^{n^2} - 1$

8. (c) 1

9. (c) $\mathbb{R}$

10. (d) $x \in [2, 4)$

11. (b) $-\frac{4}{5}$ or $\frac{4}{5}$

12. (d) $\frac{1}{2}$

13. (a) 1

14. (b) $-1+i$

15. (c) $1-i$

16. (c) $-x > -5$

17. (d) $(-\infty, -4) \cup [6, \infty)$

18. (d) If $xy < 0$, then $x < 0$ and $y < 0$

19. (A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

20. (B) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

SECTION-B

21. $R = \{x \in \mathbb{Z} : x \text{ is divisible by } 2\}$

$S = \{y \in \mathbb{Z} : y \text{ is divisible by } 3\}$

then $S \cap R = \{x : x \text{ is divisible by both } 2 \text{ and } 3\}$

22. (A) $R_1 = \{(2,3), (3,0), (2,7), (-4,6)\}$

Since 2 has two images 3 and 7.

Hence R_1 is not a function.

OR. (B) $R_2 = \{(x, x^2) : x \text{ is a real number}\}$

$\therefore$ for every $x \in \mathbb{R}$ there is a unique image $x^2 \in \mathbb{R}^+$

i.e $x \longrightarrow x^2$ and $-x \longrightarrow x^2$ for $x \in \mathbb{R}^+ \cup \{0\}$

Here (i) $\text{dom}(R_2) = \mathbb{R}$

(ii) R_2 is not one-many relation.

Hence R_2 is a function.

23. $\sin\theta = \frac{4}{5}$. then $\cos\theta = \pm\sqrt{1-\sin^2\theta} = \pm\sqrt{1-\frac{16}{25}} = \pm\frac{3}{5}$

and $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{4/5}{\pm 3/5}$

$\Rightarrow \boxed{\tan\theta = \pm\frac{4}{3}}$

24. $|1+3i| = \sqrt{1^2+3^2} = \sqrt{10}$

25. (A) $3x-5 < x+7$

$\Rightarrow 3x-x < 7+5$

$\Rightarrow 2x < 12$

$\Rightarrow x < 6$ $\quad (\because a<b \Rightarrow ac<bc \text{ if } c>0)$

$\therefore x \in (-\infty, 6)$

(B) $4x+3 \geq 2x+17$

$\Rightarrow 4x-2x \geq 17-3$

$\Rightarrow 2x \geq 14$

$\Rightarrow x \geq 7$ $\quad (\text{As } a>b \Rightarrow ac>bc \text{ if } c>0)$

$\therefore x \in [7, \infty)$

26. (i) $D = \{x \in \mathbb{R} \mid x^3 = x\}$

$\therefore x^3 - x = 0$

$\Rightarrow x(x-1)(x+1) = 0$

$\Rightarrow x = 0, 1 \text{ or } -1$

$\therefore D = \{-1, 0, 1\}$

(ii) $E = \{x \in \mathbb{R} \mid x^4 - 5x^2 + 6 = 0\}$

$\therefore x^4 - 5x^2 + 6 = 0$

$\Rightarrow (x^2-3)(x^2-2) = 0$

$\Rightarrow x = \pm\sqrt{3} \text{ or } \pm\sqrt{2}$

$\therefore E = \{-\sqrt{3}, -\sqrt{2}, \sqrt{2}, \sqrt{3}\}$

27. $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$

For equal, $f(x) = g(x)$

$\Rightarrow 2x^2 - 1 = 1 - 3x$

$\Rightarrow 2x^2 + 3x - 2 = 0$

$\Rightarrow 2x^2 + 4x - x - 2 = 0$

$\Rightarrow (2x-1)(x+2) = 0$

$\Rightarrow x = \frac{1}{2}$ or -2

$\therefore$ domain $= \{-2, \frac{1}{2}\}$, where both functions are equal.

28. $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$

then $\tan(\theta+\phi) = \dfrac{\tan\theta + \tan\phi}{1 - \tan\theta \cdot \tan\phi}$

$= \dfrac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} = \dfrac{5/6}{5/6} = 1$

$\Rightarrow \boxed{\theta + \phi = (2n+1)\pi \pm \frac{\pi}{4}}$

29. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$

$= i^n (1 + i + i^2 + i^3)$

$= i^n (1 + i - 1 - i) = 0$

30. $\dfrac{5 + \sqrt{2}i}{1 - \sqrt{2}i} = \dfrac{5 + \sqrt{2}i}{1 - \sqrt{2}i} \times \dfrac{1 + \sqrt{2}i}{1 + \sqrt{2}i}$

$= \dfrac{3 + 6\sqrt{2}i}{3} = 1 + 2\sqrt{2}i$

31. $\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$

$= \frac{25x-10-21x+9}{15}$

$\Rightarrow \frac{x}{4} < \frac{4x-1}{15}$

$\Rightarrow 15x < 16x - 4$

$\Rightarrow 4 < 16x - 15x$

$\Rightarrow \boxed{4 < x}$

$\therefore x \in (4, \infty)$

SECTION-D

32. $f(x) = \frac{1}{\sqrt{[x]^2 - [x] - 6}}$

Since $[x]^2 - [x] - 6 > 0$

$\Rightarrow ([x] - 3)([x] + 2) > 0$

$\Rightarrow [x] > 3$ and $[x] + 2 > 0$

OR

$[x] < 3$ and $[x] + 2 < 0$

$\Rightarrow [x] > 3$ and $[x] > -2$

OR

$[x] < 3$ and $[x] < -2$

$\Rightarrow [x] > 3$ OR $[x] < -2$

$\Rightarrow [x] \geq 4$ OR $[x] \leq -3$

$\Rightarrow 4 \leq x$ OR $x \leq -2$

$\Rightarrow x \in [4, \infty)$ OR $x \in (-\infty, -2)$

$\therefore x \in [4, \infty) \cup (-\infty, -2)$

33. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$

then $m^2 - n^2 = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$= 4 \tan\theta \cdot \sin\theta \quad (\text{as } (a+b)^2 - (a-b)^2 = 4ab)$

34. (A) $\overline{\left(\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}\right)} = \frac{\overline{(3-2i)(2+3i)}}{\overline{(1+2i)(2-i)}} \quad \left(\text{AS } \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}\right)$

$= \frac{\overline{(3-2i)}\ \overline{(2+3i)}}{\overline{(1+2i)}\ \overline{(2-i)}} \quad \left(\text{AS } \overline{z_1 z_2} = \overline{z_1}\ \overline{z_2}\right)$

$= \frac{(3+2i)(2-3i)}{(1-2i)(2+i)}$

$= \frac{(6-6)+i(-9+4)}{(2+2)+i(-4+1)}$

$= \frac{-5i}{4-3i}$

$= \frac{-5i}{4-3i} \times \frac{4+3i}{4+3i}$

$= \frac{15-20i}{25}$

$= \frac{3}{5} - \frac{4}{5}i$

34. (B) $\left|\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}\right|$

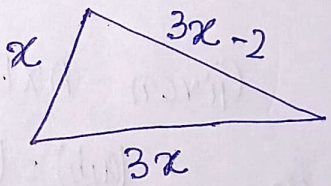
$= \frac{|(3-2i)(2+3i)|}{|(1+2i)(2-i)|}$ $\left(\text{As } \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right)$

$= \frac{|3-2i|\,|2+3i|}{|1+2i|\,|2-i|}$ $\left(\text{As } |z_1 z_2| = |z_1|\,|z_2|\right)$

$= \frac{\sqrt{13} \times \sqrt{13}}{\sqrt{5} \times \sqrt{5}}$

$= \frac{13}{5}$

35. (A) Let the shortest side $= x$ cm



ACQ Longest side $= 3x$ cm

and third side $= 3x-2$ cm

in a triangle.

Given perimeter of triangle $\geq$ 61 cm

$\Rightarrow x + 3x + 3x - 2 \geq 61$

$\Rightarrow 7x \geq 63$

$\Rightarrow \boxed{x \geq 9 \text{ cm}}$

$\therefore$ minimum length of shortest distance is 9 cm.

OR (35) In five paper marks obtained by sunita is 87, 92, 94, 95 and in 5th paper say x.

ATQ, $\frac{87+92+94+95+x}{5} \geq 90$ (as Average ≥ 90)

$\Rightarrow x + 368 \geq 90 \times 5$

$\Rightarrow x \geq 450 - 368$

$\Rightarrow \boxed{x \geq 82}$

$\therefore$ To get grade A, sunita must obtain atleast 82 in the 5th paper.

SECTION-E (Case-Studies)

36. Given $A \times B = \{(a,b) : a \in R, b \in R\}$

$(a,b) = (c,d)$ iff $a=c$ and $b=d$

(i) If $(a-3, b+7) = (3,7)$

$\therefore a-3 = 3$ and $b+7 = 7$

$\Rightarrow \boxed{a=6}$ and $\boxed{b=0}$

(ii) If $(x+6, y-2) = (0,6)$

$\therefore x+6 = 0$ and $y-2 = 6$

$\Rightarrow \boxed{x=-6}$ and $\boxed{y=8}$

(iii) If $(x+2, 4) = (5, 2x+y)$

$\therefore x+2 = 5$ and $4 = 2x+y$

$\Rightarrow \boxed{x=3}$ and $y = 4-2x \Rightarrow \boxed{y=-2}$

(iv) If $(x+3, 5) = (6, 2x+y)$

$\therefore x+3 = 6$ and $5 = 2x+y$

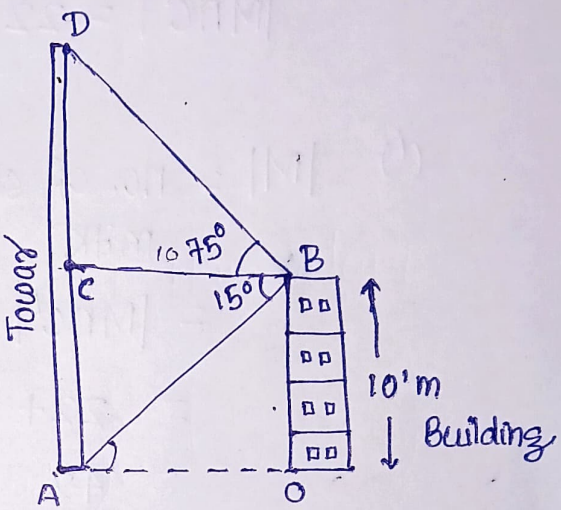
$\Rightarrow \boxed{x=3}$ and $y = 5-2x \Rightarrow \boxed{y=-1}$

37. Ans,

Height of building (h) = 10 m

Angle of elevation = $\angle CBD = 75^\circ$

angle of depression = $\angle CBA = 15^\circ$



~~In $\triangle ABC$, $\tan 15^\circ = \tan 30 + 45$~~

(i) $\tan 15^\circ = \tan(45^\circ - 30^\circ)$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ}$$

$$= \frac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

(ii) $\tan 75^\circ = \tan(30 + 45^\circ)$

$$= \frac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \cdot \tan 45^\circ}$$

$$= \frac{1/\sqrt{3} + 1}{1 - 1/\sqrt{3}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

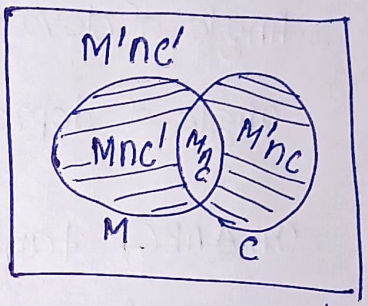
(i) ~~In $\triangle ABC$, $\tan 15^\circ = \frac{AC}{BC} = \frac{10}{\cdot}$~~

38. Given in a farewell party of 100 students, two drinks are served i.e. Milk(M) and Coffee(C).

ACQ, so, $|U| = 100$

$|M \cap C| = 20$

$|M' \cap C'| = 22$ and $|M' \cap C| = 15$



(i) $|M|$ = no. of students who prefer milk

$= |M \cap C'| + |M \cap C|$

$= 22 + 20$

$= 42$

(ii) $|M' \cap C'|$ = no. of students who doesn't prefer neither milk or coffee

$= |U| - |M \cap C'| - |M \cap C| - |M' \cap C|$

$= 100 - (22 + 20 + 15)$

$= 100 - 57$

$= 43$.