General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains **38 questions**. All questions are **compulsory**.
(ii) Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.
(iii) In Section **A** – Question Number **1** to **18** are Multiple Choice Questions (MCQs) and Question Number **19 & 20** are Assertion-Reason based questions of **1 mark** each.
(iv) In Section **B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2 marks** each.
(v) In Section **C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3 marks** each.
(vi) In Section **D** – Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5 marks** each.
(vii) In Section **E** – Question Number **36** to **38** are case study-based questions, carrying **4 marks** each.
(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, **2** questions in Section – **D** and **2** questions in Section – **E**.
(ix) Use of calculator is **NOT** allowed.

SECTION-A 1×20

This section comprises of Multiple-Choice Questions (**MCQ**s) of 1 mark each. Select the correct option (Question **1** to Question **18**):

1. What is the remainder when 5^{61} is divided by 7? **[1]**
(a) 3 (b) 4 (c) 5 (d) 6

2. In what ratio must a shopkeeper mix two types of oranges worth 60 per kg and 90 per kg respectively so as to get a mixture at 80 per kg. **[1]**
(a) $1:2$ (b) $2:1$ (c) $2:3$ (d) $3:2$

3. A man rows 15 km upstream and 25 km downstream in 5 hours each time. What is the speed of the current? **[1]**
(a) 1 $km/h.$ (b) 1.5 $km/h.$ (c) 2 $km/h.$ (d) 0.5 $km/h.$

4. A pump can fill a tank with water in 20 minutes and another pump can fill the same tank in 30 minutes. If both the pumps are opened together, then how much time will be taken to fill the tank completely? **[1]**
(a) 60 min. (b) 50 min. (c) 10 min. (d) 12 min.

5. A can run 22.5 m. while B runs 25 m in the same time. In a 1000 m race, find by how much distance B beats A? **[1]**
(a) 100 m (b) 120 m (c) 80 m (d) 150 m

6. Which of the following is true? **[1]**
(a) A.M.≥G.M. (b) A.M.≤G.M. (c) A.M.=G.M. (d) A.M.≠G.M.

7. The matrix $A=\begin{pmatrix}0 & 0 & 2\\ 0 & 2 & 0\\ 2 & 0 & 0\end{pmatrix}$ is a **[1]**
(a) Scalar matrix (b)Diagonal matrix (c) Square matrix (d) NOT

8. What is the second order derivative of x^x? **[1]**
(a) $x^x(1-logx)$ (b) $x^x(logx-1)$ (c) $x^x(2+logx)$ (d) $x^x(1+logx)$

9. If x is real, the minimum value of $x^2 - 8x + 17$ is **[1]**

(a) -1 (b) 0 (c) 1 (d) 2

10. Integration of x is **[1]**

(a) x (b) 1 (c) $\frac{x^2}{2}$ (d) $\frac{x^2}{2} + c$

11. Variance formula is **[1]**

(a) $\sum p_i x_i^2 - (\sum p_i x_i)^2$ (b) $(\sum p_i x_i)^2 - \sum p_i x_i^2$

(c) $\sqrt{\sum p_i x_i^2 - (\sum p_i x_i)^2}$ (d) $\sqrt{(\sum p_i x_i)^2 - \sum p_i x_i^2}$

12. Binomial distribution is given by **[1]**

(a) $P(X=r) = {}^nC_r\, p^r\, q^{n-r}$ (b) $P(X=r) = {}^nC_r\, p^{n-r}\, q^r$

(c) $P(X=r) = {}^nP_r\, p^r\, q^{n-r}$ (d) $P(X=r) = {}^nP_r\, p^{n-r}\, q^r$

13. Mean of binomial distribution is **[1]**

(a) np (b) nq (c) npq (d) NOT

14. A coin is tossed 2 times, then probability of getting of 2 head is **[1]**

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) 1

15. Sum of probability of an experiment is **[1]**

(a) 0 (b) 1 (c) $\frac{1}{2}$ (e) NOT

16. Time series analysis helps to **[1]**

(a) Understand the behaviour of a variable in the past

(b) Predict the future behaviour of a variable

(c) Plan future operations

(d) All of the above

17. The solution of LPP with basic feasible solutions (0, 0), (10, 0), (0, 20), (10, 15) and objective function $max\ Z = 2x + 3y$ is **[1]**

(a) $x = 0, y = 20, \max z = 60$ (b) $x = 10, y = 15, \max z = 65$

(c) $x = 10, y = 20, \max z = 70$ (d) $x = 15, y = 10, \max z = 60$

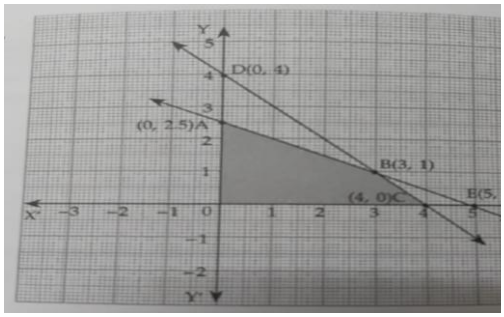
18. Besides non-negative constraints, the figure given alongside is subject to which of the following constraints? **[1]**

(a) $x + 2y \leq 5, x + y \leq 4$

(b) $x + 2y \geq 5, x + y \leq 4$

(c) $x + 2y \geq 5, x + y \geq 4$

(d) $x + 2y \leq 5, x + y \geq 4$



Assertion – Reason

Questions number **19** and **20** are Assertion – Reason based questions of **1** mark each. Two statements are given – one labelled Assertion (**A**) and other labelled Reason (**R**). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below:

(A)Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(B)Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(C)Assertion (A) is true but reason (R) is false.

(D)Assertion (A) is false but reason (R) is true.

19. Assertion (A): A man invests ₹ 25000 in ₹100 shares available at ₹125. If the dividend paid by the company is 10%, then his rate of return is 8%. **[1]**

Reason (R): Rate of return $= \frac{\text{Net profit/Loss}}{\text{Cost of investment}} \times 100$

20. Assertion (A): If the feasible region for an LPP is bounded, then the objective function $Z = ax + by$ has both maximum and minimum values. **[1]**

Reason (R): A feasible region of a system of linear inequalities is said to be bounded if it can be enclosed within a circle.

SECTION-B 2× 5

Questions Number **21** to **25** are Very Short Answer (VSA) type questions of **2** marks each.

21. Two pipes can fill a cistern in 8 and 12 hours respectively. The pipes are opened simultaneously and it takes 12 minutes more to fill the cistern due to leakage. If the cistern is full, what will be the time taken by the leakage to empty it? **[2]**

22. A person invests ₹10000 in 10% ₹100 shares of a company available at a premium of ₹25. Find his rate of return. **[2]**

23. (A) A die is rolled. If a random variable X is defined as the number on the upper face, then find its probability distribution. **[2]**

OR

(B) A die is rolled. If a random variable X is defined as the number on the upper face be even, then find its probability distribution. **[2]**

24. (A) Given that mean of the normal variate X is 9 and standard deviation is 3, then find: **[2]**

(i) The Z-score of data point 15

(ii) The data points if its Z-score is 4

OR

(B) For testing the significance of difference between the means of two independent samples, the degree of freedom (v) is taken as: **[2]**

(a) $n_1 - n_2 + 2$ (b) $n_1 - n_2 - 2$ (c) $n_1 + n_2 + 2$ (d) $n_1 + n_2 - 2$

25. The number of letters, in hundreds, posted in a certain city on each day for a week is given as follows: **[2]**

35, 70, 36, 59, 62, 60, 71.

Calculate the three-day moving averages.

SECTION-C 3× 6

Questions Number **26** to **31** are Short Answer (SA) type questions of **3** marks each.

26. (A) In a 500 m race, the ratio of speeds of two participants A and B is 4:5 respectively. If A has a start of 180 m, then find the distance by which A wins. **[3]**

OR

(B) A container has 50 litres of juice in it. 5 litres of juice is taken out and is replaced by 5 litres of water. This process is repeated 4 more times. Determine the quantity of juice in the container after final replacement. **[3]**

27. Suppose that X has a poisson distribution. If $P(X = 2) = \frac{2}{3} P(X = 1)$, then find mean of the distribution. **[3]**

28. (A) Find derivative of $\log(\log x)$ **[3]**

OR

(B) Evaluate $\int_0^1 log(1 + 2x)dx$ **[3]**

29. (A) An asset costs ₹ 4,50,000 with an estimated useful life of 5 years and a scrap value of ₹ 1,00,000. Using linear depreciation method, find the annual **[3]**

depreciation of the asset and construct a yearly depreciation schedule

OR

(B) Amrita bought a car worth ₹12,50,000 and makes a down payment of ₹ 3,00,000. The balance amount is to be paid in 4 years by equal monthly instalments at an interest rate of 15% p.a. Find the EMI that Amrita has to pay for the car. **[3]**

$\{\text{Given } (1 \cdot 0125)^{-48} = 0 \cdot 5508565)\}$

30. From the following time-series, determine the trend value by 3-yearly moving averages: **[3]**

Year	Sales (in ₹' 000)
2011	9
2012	11
2013	12
2014	15
2015	18
2016	17
2017	19

31. (A) Hole punching machine is set to punch a hole 1.84 cm in diameter in a strip of sheet metal in a manufacturing process. The strip of metal is then creased and sent on to the next phase of production, where a metal rod is slipped through the hole. It's important that the hole be punched to the specified diameter of 1.84 cm. To test punching accuracy, technicians randomly sampled 16 punched holes and measured the diameters. The sample data (in centimetres) has a mean of 1.85 and variance 0.0064. Set up null and alternate hypothesis to test if the machine is working properly (whether the holes are being punched an average of 1.84 centimetres), at an alpha level of 0.05. Assume the punched holes are normally distributed in the population. **[3]**

Given: **[** t (0.05,15) = 2.131**]**

OR

(B) The mean weekly sales of a four-wheeler were 50 units per agency in 20 agencies. After an advertising campaign, the mean weekly sales increased to 55 units per agency with standard deviation of 10 units. Test whether the advertising campaign was successful. **[3]**

(Use $t_{0 \cdot 005}$=1·729 for 19 d.f.**)**

SECTION-D

5× 4

Questions number **32** to **35** are Long Answer (LA) type questions of **5** marks each.

32. (A) If A = $\begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$ find A^{-1} and hence solve the following system of equations: **[5]**

$$2x - 3y + 5z = 11,$$
$$3x + 2y - 4z = -5,$$
$$x + y - 2z = -3$$

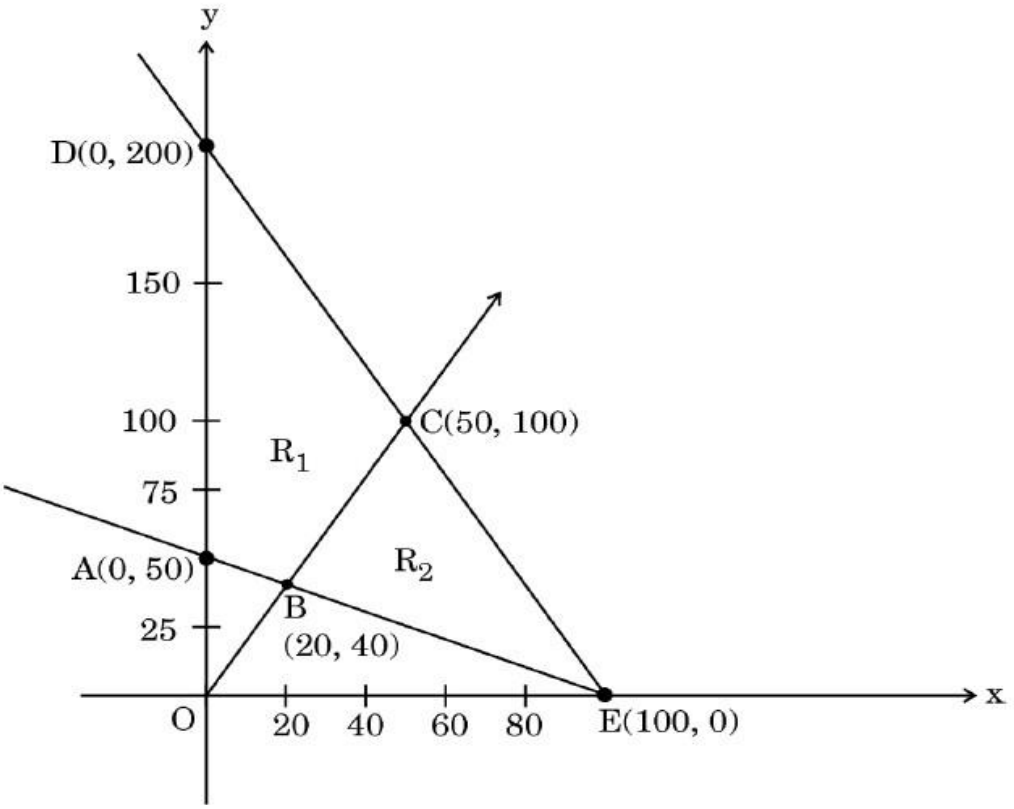
OR

(B) Using properties of determinants, prove that **[5]**

$$\Delta=\begin{vmatrix}(b+c)^2 & a^2 & a^2\\ b^2 & (c+a)^2 & b^2\\ c^2 & c^2 & (a+b)^2\end{vmatrix} = 2abc(a+b\_c)^3$$

33. In number theory, it is often important to find factors of an integer N. The number N has two trivial factors, namely 1 and N. Any other factor, if exists, is called non-trivial factor of N. Naresh has plotted a graph of some constraints (linear inequations) with points A (0, 50), B (20, 40), C (50, 100), D (0, 200) and E (100, 0). **[5]**

This graph is constructed using three non-trivial constraints and two trivial constraints. One of the non-trivial constraints is $x + 2y \geq 100$.



Based on the above information answer the following questions:

(i) What are the two trivial constraints?

(ii) If R_1 is the feasible region, then what are the other two non-trivial constraints?

(iii) If R_1 is the feasible region, then find the maximum value of the objective function $Z = 5x + 2y$

34. A machine cost is ₹1,00,000 and its effective life is estimated to be 12 years. A sinking fund is created is for replacing the machine by a new model at the end if its life time when its scrap realizes a sum of ₹5000 only. Find what amount should be set aside at the end of each year out of the profit for the sinking fund if it accumulates at 5% effective. [Use $(1,05)^{12}$=1.7958] **[5]**

35. **(A)** Evaluate $\int \frac{dx}{(1+e^x)(1+e^x)}$ **[5]**

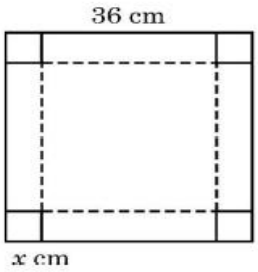
OR

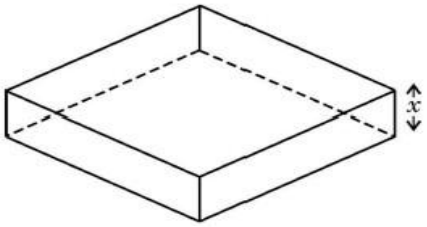
(B) $\int x\, log(1+x^2)dx$ **[5]**

SECTION-E 4× 3

Questions number **36** to **38** are case-study based questions of **4** marks each.

36. A man has an expensive squared-shaped piece of golden board of side 36 cm. He wants to turn it into a box without top by cutting a square from each corner and folding the flaps. Let x cm be the side of square, which is cut from each corner.





Based on the above information, answer the following questions. **[1+1+2]**

(i) Find the expression for the volume (V) of open box in terms of x.

(ii) Find $\frac{dv}{dx}$

(iii) **(A)** Find the value of x for which the volume (V) is maximum.

OR

(B) Find the maximum volume of the open box.

37. Rohini wants to give a rectangular plot of land for a school in her village. When she was asked to mention the dimensions of the plot, she told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area does not alter, but if its length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 sq. m. **[1+1+2]**

based on the above information, answer the following questions.

(i) Assuming x m and y m as the length and breadth of the plot respectively, write the system of linear equations in x and y.

(ii) Write the system of linear equations obtained in (i) in the matrix equation AX = B.

(iii) **(A)** Determine A^{-1}

OR

(B) Find the area of the plot.

38. In the year 2010, Mr. Aggarwal took a home loan of ₹30,00,000 from State Bank of India at 7·5% p.a. compounded monthly for 20 years. **[1+1+2]**

Based on the above information answer the following questions:

(i) Determine the EMI.

(ii) Find the principal paid by Mr. Aggarwal in the 150th instalment

(iii)(A) Find the total interest paid by Mr. Aggarwal

OR

(B) How much was paid by Mr. Aggarwal to repay the entire amount of home loan?

$[Use\ (1 \cdot 00625)^{240} = 4 \cdot 4608;\ (1 \cdot 00625)^{91} = 1 \cdot 7629]$