

General Instructions:

Read the following instructions very carefully and strictly follow them:

- (i) This Question paper contains **38 questions**. All questions are **compulsory**.
- (ii) Question paper is divided into **FIVE Sections** – Section **A, B, C, D** and **E**.
- (iii) In Section **A** – Question Number **1** to **18** are Multiple Choice Questions (MCQs) and Question Number **19 & 20** are Assertion-Reason based questions of **1 mark** each.
- (iv) In Section **B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2 marks** each.
- (v) In Section **C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3 marks** each.
- (vi) In Section **D** – Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5 marks** each.
- (vii) In Section **E** – Question Number **36** to **38** are case study-based questions, carrying **4 marks** each.
- (viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, **2** questions in Section – **D** and **2** questions in Section – **E**.
- (ix) Use of calculator is **NOT** allowed.

SECTION-A

1×20

This section comprises of Multiple-Choice Questions (**MCQs**) of 1 mark each. Select the correct option (Question 1 to Question 18):

6. If $A = \begin{pmatrix} 1 & 0 \\ 2 & 2 \end{pmatrix}$ then $\det(A)$ is

(a) 1 (b) 0 (c) 2 (d) -1

7. For two matrices $P = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$; (where Q^T is the transpose of the matrix Q), $P - Q$ is:

(a) $\begin{pmatrix} 2 & 3 \\ -3 & 0 \\ 0 & -3 \end{pmatrix}$ (b) $\begin{pmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{pmatrix}$ (c) $\begin{pmatrix} 4 & 3 \\ 0 & -3 \\ -1 & -2 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & 3 \\ 0 & -3 \\ 0 & -3 \end{pmatrix}$

8. If solving a system of linear equations in 3 variables by Cramer's rule, we get

$\Delta=0$ and at least one of $\Delta x, \Delta y, \Delta z$ is non-zero then the system of linear equations has _____ solution.

(a) No (b) Unique (c) Infinitely many (d) Trivial

9. If $p = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to

(a) 4 (b) 8 (c) 11 (d) 256

10. If A and B are two square matrices such that $B = -A^{-1}BA$, then $(A + B)^2$ is equal to

(a) 0 (b) $A^2 + B^2$ (c) $A^2 + B^2 + 2AB$ (d) $A^2 + B^2 - 2AB$

11. If two square matrices A and B are such that $|AB|=12$ and $|B|=-4$, then value of $|A|$ is:

(a) 8 (b) -8 (c) -3 (d) 16

12. The value of the determinant $A = \begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$ is

(a) 1 (b) -1 (c) i (d) $-i$

13. For a 3×3 matrix if $\text{adj } A = 2A^{-1}$, find $|3AA^T|$

(a) 108 (b) 12 (c) 54 (d) 8

14. If $x = at^2$ and $y = 2at$, then at $t=2$ the value of $\frac{d^2y}{dx^2}$ is

(a) $\frac{-1}{16a}$ (b) $\frac{-1}{16}$ (c) $\frac{1}{8a}$ (d) $\frac{-1}{4}$

15. $\int \frac{x+2}{2x^2+6x+5} dx = P \int \frac{x+2}{2x^2+6x+5} dx + \frac{1}{2} \int \frac{dx}{2x^2+6x+5}$ then the value of P is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (e) $\frac{1}{6}$

16. The order of a differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^4 + x^2 = 0$

(a) 2 and 4 (b) 2 and 1 (c) 2 and 3 (d) 3 and 3

17. If $f'(x) = \frac{1}{1-x}$, then for $x > 1$, $f(x)$ is

(a) Decreasing (b) Constant (c) Increasing (d) Neither increasing nor decreasing

18. Integrating factor of the Differential equation $x \frac{dy}{dx} + y = 3x^2$ is

(a) x (b) $\log x$ (c) $\frac{1}{x}$ (d) NOT

Questions number **19** and **20** are Assertion – Reason based questions of **1** mark each. Two statements are given – one labelled Assertion (A) and other labelled Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below:

- (A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
- (B) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
- (C) Assertion (A) is true but reason (R) is false.
- (D) Assertion (A) is false but reason (R) is true.

19. Assertion (A): Let A be a 2×2 matrix, then $\text{adj}(\text{adj } A) = A$

Reason (R): $|\text{adj } A| = |A|$

20. Assertion (A): $y = 5e^x + 4$ is the solution of the differential equation $\frac{dy}{dx} = y - 4$

Reason (R): $\frac{d}{dx} e^x = e^x$

SECTION-B

2×5

Questions Number **21** to **25** are Very Short Answer (VSA) type questions of **2** marks each.

21. If $\begin{pmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{pmatrix}$ is skew-symmetric matrix, then find value of $a + b + c$.

22. Without finding the values of the square roots, prove that the inequality

$$\sqrt{5} + \sqrt{3} > \sqrt{6} + \sqrt{2}$$

23. Is the following relation a function? Justify your answer

(A) Find adjoint of the matrix $\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$

OR

(B) Find inverse of the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

24. Two athletes Vijay and Samuel finish 100 meters race in 12 secs and 16 secs respectively. By how many meters does Vijay defeat Samuel?

- (a) 10.2 meters
- (b) 15 meters
- (c) 25 meters
- (d) 33.3 meters

OR

In a 100m race, A can give B a start of 10 m and can give C a start of 28 m. In the same race, B can give C a start of

- (a) 10m
- (b) 20m
- (c) 18m
- (d) 8m

25. (A) What is the derivative of $e^{\log x}$

OR

(B) Find the integration of $\int_0^1 x dx$

SECTION-C**3× 6**

Questions Number **26** to **31** are Short Answer (SA) type questions of 3 marks each.

26. Find the remainder when 5^{61} is divided by 7.

OR

Find $(205 \times 604) \bmod 3$

27. Cost of two toys A and B are 50 and 75. On a particular Sunday shopkeeper P sells 7 toys of type A and 10 toys of type B whereas shopkeeper Q sells 8 toys of type A and 6 toys of type B. Find income of both shopkeepers using matrix Algebra.

28. Solve $\int xe^{x^2} dx$

OR

Solve $\int_1^2 \log x dx$

29. Find $\frac{dy}{dx} + \frac{x}{y} = 0$

OR

Find $\frac{dy}{dx} + xy = 0$

30. Find the tangent of $y^2 = x$ at (1,2)

31. Find the maximum or minimum value of $x^2 - x + 1$

SECTION-D**5× 4**

Questions number **32** to **35** are Long Answer (LA) type questions of 5 marks each.

32. (A) The purchase officer of a pharmaceutical company informs the production manager that during the month, following supply of three chemicals i.e., Aspirin (A), Caffeine (C) and Decongestant (D) used in the production of three types of pain-killing tablet will be 16 kg, 10 kg and 16 kg respectively. According to the specification, each strip of 10 tablets of Paingo requires 2 gm of A, 3 gm of C and 1 gm of D.

The requirements for other tablets are:

X-prene	4 gm of A	1 gm of C	3 gm of D
Relaxo	1 gm of A	2 gm of C	3 gm of D

Taking suitable variables form the system of linear equations and use matrix method to find the number of strips of each type so that the raw materials are consumed entirely.

OR

(B) Given the following equations for two related markets A and B, find the equilibrium conditions for each market and the price for each by Cramer's rule:

$$x_d(A) = 82 - 3p_A + p_B$$
$$x_d(A) = 92 + 2p_A - 4p_B$$

$$x_s(A) = -5 + 15p_A$$
$$x_s(B) = -6 + 32p_B$$

where x_d and x_s denotes the quantity demanded and quantity supplied respectively and p_A and p_B represents the price for each in the two markets.

33. Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$

34. Evaluate $\int_0^1 \frac{\log x}{\log x + \log(1-x)} dx$

35. Two pipes can fill a cistern in 8 and 12 hours respectively. The pipes are opened simultaneously and it takes 12 minutes more to fill the cistern due to leakage. If the cistern is full, what will be the time taken by the leakage to empty it?

OR

The speed of a boat in still water is 14 km per hour. While going downstream it moves at the rate of 24 km per hour. Find the speed of the boat against the stream.

SECTION-E

4×3

Questions number 36 to 38 are case-study based questions of 4 marks each.

36. A, B and C are three pipes connected to a tank. A and B together fill the tank in 6 hours. B and C together fill the tank in 6 hours. B and C together fill the tank in 10 hours. A and C together fill the tank in $7\frac{1}{2}$ hours.

Based on the above information, answer the following questions.

- (i) In how much time will A, B and C fill the tank.
- (ii) In how much time will A separately fill the tank?
- (iii) (A) In how much time will B separately fill the tank?

OR

(B) In how much time will C separately fill the tank?

37. Amit, Biraj and Chirag were given the task of creating a square matrix of order 2. Below are the matrices created by them. A, B, C are the matrices created by Amit, Biraj and Chirag respectively

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}, B = \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix} \text{ and } C = \begin{pmatrix} 2 & 0 \\ 1 & -2 \end{pmatrix}$$

If $a=4$ and $b=-2$ based on the above information, answer the following questions.

- (a) Find the sum of the matrices A, B and C.
- (b) Find $(A^T)^T$.

(c) Find $AC-BC$

38. Two schools P and Q decided to award their selected students for the values of discipline and honesty in the form of prizes at the rate of $\text{₹}x$ and $\text{₹}y$ respectively. School P decided to award respectively 3, 2 students a total prize money of $\text{₹}2300$ and school Q decided to award respectively 5, 3 students a total prize money of $\text{₹}3700$.

Based on the above information answer the following questions:

- (i) Write the matrix equation representing the above situation.
- (ii) Find the value of the determinant of coefficients of x and y.
- (iii) (A) Find the values of x and y respectively (use Cramer's rule).

OR

- (B) Find the inverse of the matrix.