

CLASS - XI

1.

Given $L_1: ax + by = c$ and $L_2: a'x + b'y = c'$ are perpendicular.

Slope of $L_1 = m_1 = -\frac{a}{b}$

Slope of $L_2 = m_2 = -\frac{a'}{b'}$

Since $L_1 \perp L_2 \Rightarrow m_1 m_2 = -1$

$\Rightarrow -\frac{a}{b} \times -\frac{a'}{b'} = -1$

$\Rightarrow \boxed{aa' + bb' = 0}$

$\therefore$ option (a)

2.

Distance of the point $P(1, -3)$ from the line $2y - 3x = 4$

is $d = \frac{|2(-3) - 3(1) - 4|}{\sqrt{2^2 + (-3)^2}} = \frac{13}{\sqrt{13}} = \sqrt{13}$

$\therefore$ option (c)

3.

Let AB straight line
intercept at $A = (x_0, 0)$
and $B = (0, y_0)$ and $P(1, 2)$
bisect the line segment AB.

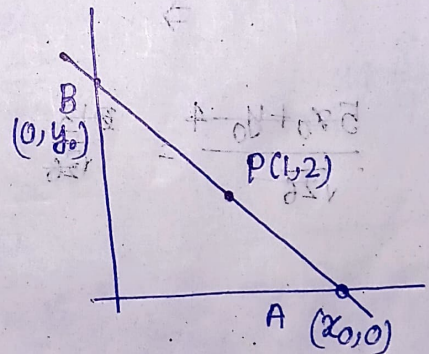
Then by mid-point formula

$\frac{x_0 + 0}{2} = 1$ and $\frac{0 + y_0}{2} = 2$

$\Rightarrow \boxed{x_0 = 2}$ and $\boxed{y_0 = 4}$

Then by intercept form equation of the line is

$\frac{x}{2} + \frac{y}{4} = 1 \Rightarrow \boxed{2x + y = 4} \therefore$ option (d)

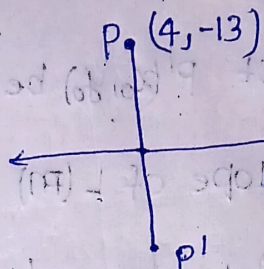


[4.]

$$P = (4, -13) \text{ and } L: 5x + y + 6 = 0$$

$$\text{Slope of } L: -5$$

$$\therefore \text{slope of } PP' = \frac{1}{\text{slope of } L} = \frac{1}{-5}$$



$$\text{Slope of } (4, -13) \text{ and } (-1, -14) \text{ is } \frac{-14 - (-13)}{-1 - 4} = \frac{-1}{-5} = \frac{1}{5}$$

$$\text{Slope of } (4, -13) \text{ and } (3, 4) \text{ is } \frac{4 - (-13)}{3 - 4} = \frac{17}{-1} = -17$$

$$\text{Slope of } (4, -13) \text{ and } (0, 9) \text{ is } \frac{9 - (-13)}{0 - 4} = \frac{22}{-4} = -\frac{11}{2}$$

$$\text{Slope of } (4, -13) \text{ and } (1, 2) \text{ is } \frac{2 - (-13)}{1 - 4} = \frac{15}{-3} = -5$$

Hence reflection point is $(-1, -14)$

$\therefore$ option (a)

[5.]

option (d)

Section-B

[6.]

Slope of the line passing through $(3, -2)$ and $(-1, 4)$ is

$$\frac{4 - (-2)}{-1 - 3} = \frac{6}{-4} = -\frac{3}{2}$$

[7.]

Slope of $x + y + 7 = 0$ is $m_1 = -1$

Slope of 1^{st} line to it is $m_2 = \frac{-1}{m_1} = 1$

Eqⁿ of line passing through $(1, 2)$ and having

slope m_2 is

$$y - 2 = m_2(x - 1)$$

$$\Rightarrow y - 2 = 1(x - 1)$$

$$\Rightarrow \boxed{y - x - 1 = 0}$$

8.

Slope of the line $L: x+y+7=0$

is $m_1 = -1$

slope of L_2 (per to L) is $m_2 = -\frac{1}{m_1}$

$$\Rightarrow \boxed{m_2 = 1}$$

R divide AB in 1:n ratio, where $A=(1,0)$ and $B=(2,3)$

$$\therefore R = \left(\frac{2+n}{1+n}, \frac{3}{1+n} \right)$$

1. By slope-point form eqⁿ of the line passing through

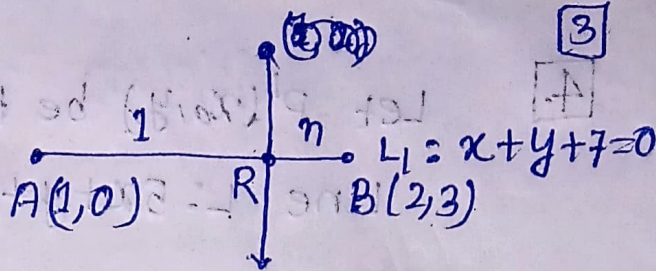
R and having slope m_2 is

$$y - \frac{3}{1+n} = m_2 \left(x - \frac{2+n}{1+n} \right)$$

$$\Rightarrow (1+n)y - 3 = 1 \left((1+n)x - (2+n) \right)$$

$$\Rightarrow \boxed{(1+n)y - (1+n)x - 1 + n = 0}$$

3



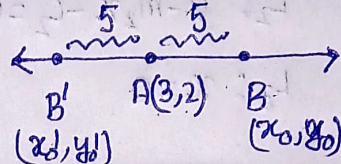
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Eqⁿ of the line passing through the points $A(3,2)$ and having slope $\frac{3}{4}$ is

$$y-2 = \frac{3}{4}(x-3)$$

$$\Rightarrow \boxed{4y-3x+1=0} \quad \text{--- (i)}$$

Let $B = (x_0, y_0)$ and $B' = (x'_0, y'_0)$ are two points on the line L such that $AB = AB'$.



By mid-point formula $\frac{x_0+x'_0}{2} = 3$ and $\frac{y_0+y'_0}{2} = 2$

$$\Rightarrow \boxed{x_0+x'_0=6} \text{ and } \boxed{y_0+y'_0=4}$$

Since $AB = AB' = 5$

$$\Rightarrow (x'_0-3)^2 + (y'_0-2)^2 = (x_0-3)^2 + (y_0-2)^2 = 25 \quad \text{--- (ii)}$$

Consider $(x_0-3)^2 + (y_0-2)^2 = 25$ --- (iii)

$$\text{At } (x_0, y_0) \quad 4y_0 - 3x_0 + 1 = 0$$

$$\Rightarrow \boxed{y_0 = \frac{3x_0-1}{4}}$$

From (iii)

$$(x_0-3)^2 + \left(\frac{3x_0-1}{4}-2\right)^2 = 25$$

$$\Rightarrow 16(x_0^2 + 9 - 6x_0) + (9x_0^2 + 81 - 54x_0) = 400$$

$$\Rightarrow 25x_0^2 - 150x_0 - 175 = 0$$

$$\Rightarrow x_0^2 - 6x_0 - 7 = 0$$

$$\Rightarrow (x_0-7)(x_0+1) = 0$$

$$\Rightarrow \boxed{x_0=7} \text{ or } \boxed{x_0=-1}$$

$$\therefore \text{At } x_0 = -1 ; y_0 = \frac{3(-1)-1}{4} = -1$$

$$\text{At } x_0 = 7 ; y_0 = \frac{3(7)-1}{4} = 5$$

$\therefore$ points are $(-1, -1)$ and $(7, 5)$

10.

$$L_1: y - m_1x - c_1 = 0$$

$$L_2: y - m_2x - c_2 = 0$$

$$L_3: x = 0$$

To Find A

$$L_3: x = 0$$

$$L_2: y - m_2x - c_2 = 0$$

$$\therefore \boxed{y = c_2} \text{ and } \boxed{x = 0}$$

$$\therefore A(0, c_2)$$

To Find C

$$L_3: x = 0$$

$$L_1: y - m_1x - c_1 = 0$$

$$\therefore \boxed{y = c_1} \text{ and } \boxed{x = 0}$$

$$\therefore C = (0, c_1)$$

To Find B

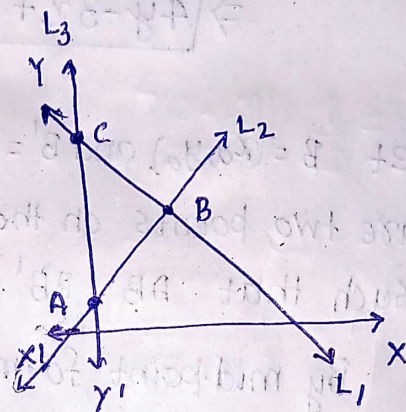
$$L_1: y - m_1x - c_1 = 0$$

$$L_2: y - m_2x - c_2 = 0$$

$$\therefore y = \frac{(m_1)(-c_2) - (m_2)(-c_1)}{(1)(m_2) - (1)(m_1)} = \frac{m_1c_2 - m_2c_1}{m_1 - m_2}$$

$$x = \frac{(-c_1)(1) - (-c_2)(1)}{1(m_2) - 1(-m_1)} = \frac{c_2 - c_1}{m_1 - m_2}$$

$$\therefore B = \left(\frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1c_2 - m_2c_1}{m_1 - m_2} \right)$$



$$\Rightarrow \sqrt{3} - \sqrt{3} m_2 = 1 + m_2$$

$$\text{or } \sqrt{3} - \sqrt{3} m_2 = -1 - m_2$$

$$\Rightarrow \sqrt{3} - 1 = m_2(1 + \sqrt{3})$$

$$\text{or } m_2 = \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$$

Area of the ΔABC

$$A = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

$$= \frac{1}{2} \left| 0(c_1 - \frac{m_1 c_2 - m_2 c_1}{m_1 - m_2}) + 0(\frac{m_1 c_2 - m_2 c_1}{m_1 - m_2} - c_2) + \frac{c_2 - c_1}{m_1 - m_2}(c_2 - c_1) \right|$$

$$= \frac{1}{2} \left| \frac{(c_2 - c_1)^2}{m_1 - m_2} \right|$$

$$\Rightarrow A = \frac{|c_2 - c_1|^2}{2|m_1 - m_2|}$$

Method-I

(B) Eqⁿ of the line

$$L: x + y - 4 = 0$$

$$\text{Slope of } L_1 = m_1 = \tan \theta_1 = -1$$

$$\Rightarrow \theta_1 = \frac{3\pi}{4}$$

Distance from a point $C(1, 2)$
from L is

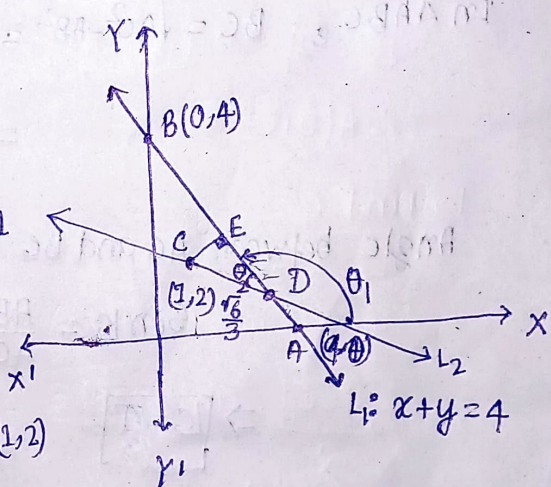
$$CE = \frac{|1 + 2 - 4|}{\sqrt{1^2 + 1^2}} = \frac{1}{\sqrt{2}}$$

$$\text{and given } CD = \frac{\sqrt{6}}{3}$$

$$\text{In } \Delta CDE, \tan \theta_2 = \frac{CE}{CD} = \frac{1/\sqrt{2}}{\sqrt{6}/3} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta_2 = \frac{\pi}{3}$$

Angle made by L_2 with x -axis $= \theta_1 + \theta_2$
 $\therefore$ Slope of line $(L_2) = \tan(\theta_1 + \theta_2)$ or $\tan(\theta_1 - \theta_2)$
 $= \tan\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$ or $\tan\left(\frac{3\pi}{4} - \frac{\pi}{3}\right)$



$$= \tan(135^\circ + 60^\circ) \text{ or } \tan(135^\circ - 60^\circ)$$

$$= \tan 195^\circ \text{ or } \tan 75^\circ$$

$$= \tan 15^\circ \text{ or } \tan 75^\circ$$

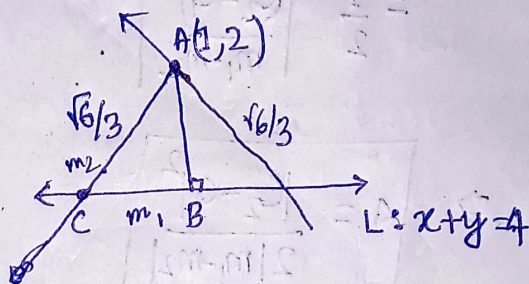
$$= 2 - \sqrt{3} \text{ or } 2 + \sqrt{3}$$

Method-II

Eqⁿ of the line $L: x+y=4$

Distance of the point $A(1,2)$

$$\text{from } L = AB = \frac{|1+2-4|}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}}$$



$$\text{Given } AC = \sqrt{6}/3$$

$$\begin{aligned} \text{In } \triangle ABC, \quad BC &= \sqrt{AC^2 - AB^2} = \sqrt{\left(\frac{\sqrt{6}}{3}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{6}{9} - \frac{1}{4}} = \sqrt{\frac{2}{3} - \frac{1}{4}} \\ &= \sqrt{\frac{8-3}{12}} = \sqrt{\frac{5}{12}} = \frac{1}{2} \sqrt{\frac{5}{3}} = \frac{\sqrt{5}}{2\sqrt{3}} \end{aligned} \quad (8)$$

Angle between AC and BC is

$$\sin C = \frac{AB}{AC} = \frac{1/\sqrt{2}}{\sqrt{6}/3} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \boxed{C = \frac{\pi}{3}}$$

$\therefore$ Slope of $L \neq m_1 = -1$

Let Slope of $AC = m_2$

Angle between BC and AC is $\frac{\pi}{3}$

$$\text{So } \tan \frac{\pi}{3} = \pm \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\Rightarrow \sqrt{3} = \pm \frac{(-1) - m_2}{1 - m_2} = \mp \frac{1 + m_2}{1 - m_2}$$

$$\Rightarrow \boxed{\sqrt{3} = \frac{1 + m_2}{1 - m_2}} \text{ or } \boxed{\sqrt{3} = -\frac{1 + m_2}{1 - m_2}} \text{ (not possible)}$$

$$\Rightarrow \sqrt{3} - \sqrt{3} m_2 = 1 + m_2$$

$$\text{or } \sqrt{3} - \sqrt{3} m_2 = -1 - m_2$$

$$\Rightarrow \sqrt{3} - 1 = m_2(1 + \sqrt{3})$$

$$\text{or } \sqrt{3} + 1 = (\sqrt{3} - 1) m_2$$

$$\Rightarrow m_2 = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$\text{or } m_2 = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$\Rightarrow m_2 = \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - 1^2}$$

$$\text{or } m_2 = \frac{(\sqrt{3} + 1)^2}{(\sqrt{3})^2 - 1^2}$$

$$= \frac{4 - 2\sqrt{3}}{2}$$

$$= \frac{4 + 2\sqrt{3}}{2}$$

$$\Rightarrow \boxed{m_2 = 2 - \sqrt{3}}$$

$$\Rightarrow \boxed{m_2 = 2 + \sqrt{3}}$$

4.

$$\text{Slope of AO} = \frac{-1}{\text{Slope of L}}$$

$$\Rightarrow \frac{b+13}{a-4} = \frac{-1}{-5}$$

$$\Rightarrow 5b + a + 69 = 0 \quad \text{--- (i)}$$

$$\text{Again } 5a + b + 6 = 0 \quad \text{--- (ii)}$$

$$\text{Eqn (i)} \times 5: 25b - 5a + 345 = 0$$

$$\text{Eqn (ii)} \times 1: 5a + b + 6 = 0$$

$$26b + 351 = 0$$

$$\Rightarrow b = -\frac{351}{26} \Rightarrow \boxed{b = -\frac{27}{2}}$$

$$\therefore a = 5\left(-\frac{27}{2}\right) + 69 = \frac{-135 + 138}{2} \Rightarrow \boxed{a = \frac{3}{2}}$$

$\therefore O = \left(\frac{3}{2}, -\frac{27}{2}\right)$ and it is mid-point of AB.

$$\text{So } \frac{x_0 + 4}{2} = \frac{3}{2} \quad \text{and} \quad \frac{y_0 + 13}{2} = -\frac{27}{2}$$

$$\Rightarrow \boxed{x_0 = -1} \quad \text{and} \quad \boxed{y_0 = -14}$$

Hence reflection point is $B = (-1, -14)$

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