MATHEMATICS

Q1. $-47 \pmod 9$

$= -2 \pmod 9 = 7$

$= 7$

Grace mark those have attained.

Q2. If $a > b$ and $c < 0$

then $a - c > b - c$

but as $c < 0$, $b - c > b + c$

so $a - c > b + c$

$\therefore$ option (D)

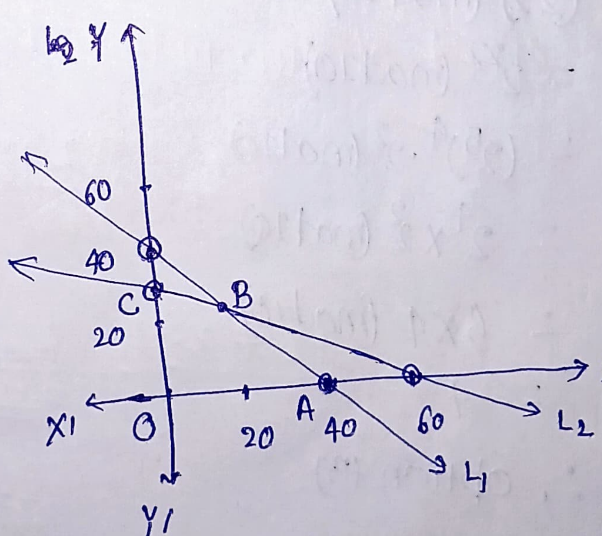
Q3. $L_1: x + y = 40$;

x	0	40
y	40	0

$L_2: x + 2y = 60$

x	0	60
y	30	0

$x, y \geq 0$



Here $A = (40, 0)$, $C = (0, 30)$, $O = (0, 0)$, and $B = (20, 20)$

For B $\quad L_1: x + y = 40$

$L_2: x + 2y = 60$

$\Rightarrow \boxed{y = 20}$

and $\boxed{x = 20}$

(x, y)	$Z = 3x + 4y$
$(0, 0)$	$Z = 0$
$(0, 30)$	$Z = 120$
$(40, 0)$	$Z = 120$
$(20, 20)$	$Z = 140$

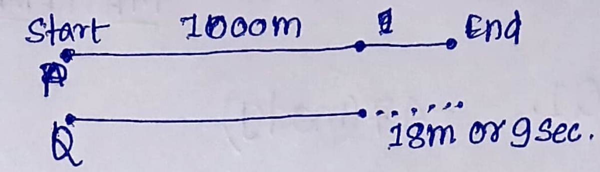
$\therefore \max(Z) = 140$

$\therefore$ option (B)

Grace for attained

Q4.

Distance to cover = 1000m



P beat B by 18'm or 9's

i.e. speed of Q $(V_Q) = \frac{18\,m}{9\,s} = 2\,m/s$

~~Distance~~ Q cover the distance when P ~~reached~~ coverd $= 1000 - 18$

$= 982\,m$

time taken by Q's to reach 982 m

$= \frac{\text{Distance}}{\text{Speed}} = \frac{982\,m}{2\,m/s} = 491\,s$

Hence P's time to complete 1000m is 491 seconds.

∴ option (D)

Q5.

$(22)^{22} \pmod{10}$

$= 2^{22} \pmod{10}$

$= (2^5)^4 \cdot 2^2 \pmod{10}$

$= 2^4 \times 2^2 \pmod{10}$

$= 6 \times 4 \pmod{10}$

$= 4$

∴ option (B)

Section-B

Q6.

Pipe A in 1 hr fill 100% of the tank

⇒ ,, ,, 1 min ,, $\left(\frac{1}{60}\right)^{th}$ portion of the tank

Pipe B in 90 min fill 100% of the tank

$\Rightarrow$ " " 1 min fill $\left(\frac{1}{90}\right)^{th}$ portion of the tank.

$\therefore$ When both simultaneous pipe open then

~~they fill~~ in 1 min they fill $\left(\frac{1}{60}+\frac{1}{90}\right)^{th}$ portion of tank

i.e. $= \left(\frac{3+2}{180}\right)^{th}$ "

$= \left(\frac{1}{36}\right)^{th}$ "

$\Rightarrow \frac{1}{36}^{th}$ part fill in 1 min

$\Rightarrow$ Full tank fill in 36 min.

Q7. $\frac{x+3}{x+2} \leq 2 \Rightarrow$

$\Rightarrow \cancel{\frac{x+3}{x+2}-2 \leq 0} \Rightarrow 1+\frac{1}{x+2} \leq 2$

$\Rightarrow \frac{1}{x+2} \leq 1$

$\Rightarrow x+2 \geq 1$

$\Rightarrow \boxed{x \geq -1}$

~~Q8.~~

Section-C

Q8. Speed of boat $= x$ (in steady water)
Speed of stream $= y$

$\therefore$ Speed of upstream $= (x-y)$ km/h
Speed of downstream $= (x+y)$ km/h

Distance travel in boat upstream and return downstream

$= 3.5 + 3.5 = 7$ Km

Distance = speed x time

(U_d) Upstream distance = Upstream speed (U_s) x Upstream time (U_t)

(D_d) Downstream distance = Downstream speed (D_s) x Downstream time (D_t)

ACQ

$$\frac{U_d}{U_s} + \frac{D_d}{D_s} = U_t + D_t$$

$$\Rightarrow \frac{3.5}{x-y} + \frac{3.5}{x+y} = 72 \text{ min.} = 1\frac{1}{5} \text{ hr} = \frac{6}{5}$$

$$\Rightarrow 3.5\left(\frac{1}{x-y} + \frac{1}{x+y}\right) = \cancel{72}\ \frac{6}{5}$$

Speed of current (y) = 1 km/h

$$\Rightarrow 3.5\left(\frac{1}{x-1} + \frac{1}{x+1}\right) = \cancel{72}\ \frac{6}{5}$$

$$\Rightarrow 3.5\left(\frac{x+1+x-1}{x^2-1}\right) = \frac{\cancel{72}}{\cancel{35}} = \frac{6}{5}$$

~~720~~ ~~144~~

$$\Rightarrow \frac{7x}{x^2-1} = \frac{6}{5}$$

$$\Rightarrow 35x = 6x^2 - 6$$

$$\Rightarrow 6x^2 - 35x - 6 = 0$$

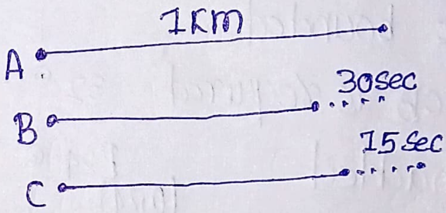
$$\Rightarrow 6x^2 - 36x + x - 6 = 0$$

$$\Rightarrow (x-6)(6x+1) = 0$$

$$\Rightarrow \boxed{x = 6 \text{ km/h}}$$

Hence speed of the boat on still water is 6 km/hr.

Q9) In a 1000 m race A beat B by 30 s, B beat C by 15 s and A beat C by 180 m.



Let time took by A to cover 1000 m = t second

then ,, ,, B ,, ,, $= (t+30)$ sec

and ,, ,, C ,, ,, $= (t+45)$ sec

Again A beat C by 180 m or 45 s

then Speed of C $(V_c) = \frac{180}{45} = 4$ m/s

$\therefore$ time took by C to cover 1000 m $= \frac{\text{Distance}}{\text{Speed}} = \frac{1000 \text{ m}}{4 \text{ m/s}}$

$= 250$ s

$\therefore$ ~~time took by A to cover 1000 m = 25~~

$$\Rightarrow t + 45 = 250$$

$$\Rightarrow \boxed{t = 205 \text{ s}}$$

10. Maximum $Z = 300x + 190y$

s.t.c. $x + y \le 24$

$2x + y \le 32$

$x, y \ge 0$

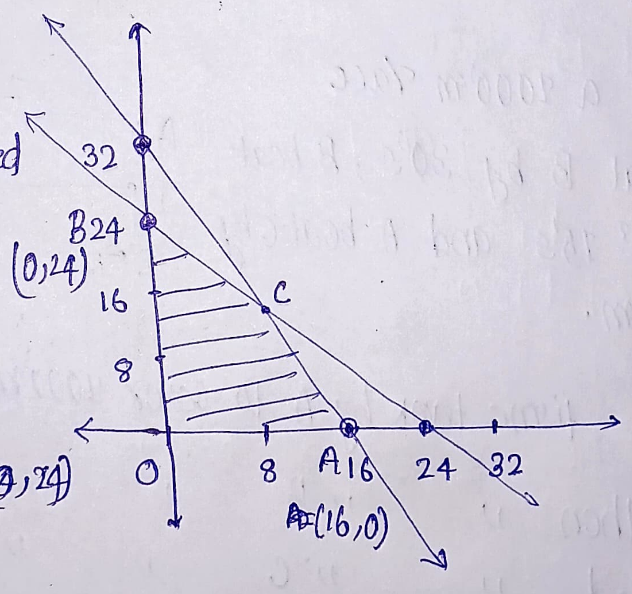
Consider $L_1: x + y = 24$

x	0	24
y	24	0

$L_2: 2x + y = 32$

x	0	16
y	32	0

Here the bounded region OACB is required ~~bounde~~ shadded feasible region.



Corner points are

$O = (0,0)$, $A = (16,0)$, $B = (0,24)$ and C.

To find C

$L_1: x + y = 24$

$L_2: 2x + y = 32$

$\Rightarrow \boxed{x = 8}$ and $\boxed{y = 16}$

$\therefore C = (8,16)$.

corner point	$Z = 300x + 190y$
(0,0)	$Z = 0$
(16,0)	$Z = 4800$
(0,24)	$Z = 4560$
(8,16)	$Z = 5440$.

$\therefore$ max $Z = 5440$ at $(8, 16)$.