Assignment-1 (Sequence & Series)

1. ~~Given~~ Let 1st term $= a$

common ratio $= r$

nth term of G.P. is $t_n = ar^{n-1}$

Since $t_4 = ar^3 = 54$

$t_9 = ar^8 = 13122$

now $\frac{ar^8}{ar^3} = \frac{13122}{54} = \frac{2\times3^8}{2\times3^3} = 3^5$

$\Rightarrow r^5 = 3^5 \Rightarrow \boxed{r=3}$

now $t_4 = 54 \Rightarrow a = \frac{54}{27} = 2$

$\therefore t_6 = ar^5$

$= 2\times3^5 = 486.$

option (c)

2. nth term of GP $t_n = ar^{n-1} = 5\times2^{n-1}$

Since $a=5$, $r=2$

For $5\times2^{n-1} = 5120$

$= 5\times1024 = 5\times2^{10}$

$\therefore n-1 = 10 \Rightarrow \boxed{n=11}$

option (c)

3. $t_n = 3\,2^{n-1}$ $\quad t_n = 12288$

From end 8th term = from start $(n-7)$th term

$= t_{n-7} = 3\,2^{n-7-1}$

$= 3\times2^{n-8} = \frac{3\times2^{n-1}}{2^7}$

option (a) $\quad = \frac{12288}{2^7} = \frac{3\times2^{12}}{2^7} = 3\times2^5 = 96$

4. $t_n = 2^n$ and $S_n = a\left(\frac{r^n - 1}{r-1}\right)$

Here $a = 2$; $r = 2$

so $S_6 = \sum_{n=1}^{6} t_n = 2\left(\frac{2^6-1}{2-1}\right) = 2 \times 63 = 126$

option (b)

5. In G.P. $a = 3$, $a_n = 3r^{n-1} = 96$

$\Rightarrow r^{n-1} = 32 = 2^5 \cancel{\Rightarrow r = 2 \;\&\; n = 6}$

$S_n = a\left(\frac{r^n-1}{r-1}\right) \Rightarrow 189 = \frac{3(r^n-1)}{(r-1)}$

$\Rightarrow \cancel{189r - 189}\ 63r - 63 = r^n - 1 = 32r - 1$

$\Rightarrow 31r = 62 \Rightarrow \boxed{r=2}$ so $n - 1 = 5 \Rightarrow \boxed{n=6}$

option (c)

6. $a = 2, r = 3$ & $S_n = a\left(\frac{r^n-1}{r-1}\right) = 3^n - 1$

$\Rightarrow 728 = 3^n - 1 \Rightarrow 3^n = 729 = 3^6 \Rightarrow \boxed{n=6}$

option (a)

7. $S_\infty = 8$, $t_2 = ar = 2$

$\Rightarrow a\left(\frac{1}{1-r}\right) = 8 \Rightarrow ar\left(\frac{1}{1-r}\right) = 8r$

$\Rightarrow 1 = 4r - 4r^2$

$\Rightarrow 4r^2 - 4r + 1 = 0 \Rightarrow (2r-1)^2 = 0 \Rightarrow \boxed{r = +1/2}$

Then $ar = 2 \Rightarrow \boxed{a = +4}$

Hence option (a)

$\boxed{8.}$ $S_\infty = \boxed{\frac{a}{1-r} = 15}$ and $t_1^2+t_2^2+\dots = \sum_{n=1}^{\infty} t_n^2 = 45$

$\Rightarrow a^2 + a^2r^2 + a^2r^4 + \dots = 45$

$\Rightarrow a^2 \frac{1-\cancel{(r^2)}}{1-r^2} = 45$

$\Rightarrow \frac{a}{1+r}\,\frac{a}{1-r} = 45 \Rightarrow \boxed{\frac{a}{1+r} = 3}$

$\frac{\frac{a}{1-r}}{\frac{a}{1+r}} = \frac{15}{3} = 5 \Rightarrow \frac{1+r}{1-r} = 5 \Rightarrow 5-5r = 1+r$

$\Rightarrow 6r = 4 \Rightarrow \boxed{r = 2/3}$

$\therefore a = 3(1+r) = 3+2 = 5$

option (c)

$\boxed{9.}$ (A) $\frac{3}{\sqrt5} + \frac{4}{\sqrt5} + \sqrt5 + \dots$ 25 terms

$= \frac{1}{\sqrt5}(3+4+5+\dots+\cancel{27}) = \frac{1}{\sqrt5}[(1+2+3+\dots+\cancel{77})-3]$

$= \frac{1}{\sqrt5}\left(\frac{\cancel{27}\times\cancel{28}}{2}-3\right) = \frac{\sqrt5}{5}(\cancel{27}\times\cancel{14}-3) = \frac{\sqrt5}{5}(\cancel{375}) = \cancel{600\sqrt5}$

$= 75\sqrt2$

So $\frac{3}{\sqrt5} + \frac{4}{\sqrt5} + \sqrt5 + \dots$ 25 term $\cancel{=}$ $75\sqrt5$

~~Hence option~~

(B) $\cancel{27}, x, 3$ are in GP

Since $\frac{x}{27} = \frac{3}{x} \Rightarrow x^2 = 3^4 \Rightarrow x = \pm 9$

~~option b~~

$\boxed{10.}$ option (c)

64
64
256
384
4096
1444$\frac{4}{7}$ 14

10. (A) $4+16+64+\cdots 6^{th}$ terms

$$= 4\left(\frac{4^6-1}{4-1}\right) = \frac{4}{3}(4096) = \cancel{4\times 136}$$

$$= \frac{4}{3}(4095) = 4\times 1365 = 5460$$

(B) $a+ar+\cdots+ar^{n-1} = a\left(\frac{r^n-1}{r-1}\right)$ iff $r\neq 1$.

11. Let $a=1$ & $b=243$

Since $a, ar, ar^2, ar^3, ar^4, ar^5 = b$

where $r = \left(\frac{b}{a}\right)^{1/5} = \left(\frac{243}{1}\right)^{1/5} = 3$

So $1, 3, 3^2, 3^3, 3^4$ and 3^5 are G.P.

12. Since infinite in G.P $p+1+\frac{1}{p}+\cdots = \frac{25}{4}$

where 1^{st} term $(a) = p$

Common Ratio $(r) = 1/p$

$$\Rightarrow \frac{a}{1-r} = \frac{25}{4}$$

$$\Rightarrow \frac{p}{1-1/p} = \frac{25}{4} \Rightarrow \frac{p^2}{p-1} = \frac{25}{4}$$

$\Rightarrow \cancel{p^3 - 25p + 25 = 0}$

~~Let $p = 1/t$~~ $\Rightarrow$?

$\Rightarrow 4p^2 = 25p - 25 = 0 \quad \Rightarrow 4p^2 - 25p + 25 = 0$

$\Rightarrow 4p^2 - 20p - 5p + 25 = 0 \Rightarrow (4p-5)(p-5) = 0$

$\therefore p = 5$ or $5/4$

$\boxed{13.}$ Since a, b, c and d are in GP

then $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$ i.e $b^2 = ac$, $c^2 = bd$ & $bc = ad$.

now $\cancel{(b+c)^2}$ $(a+b)(c+d) = ac + ad + bc + bd$

$= b^2 + bc + bc + c^2$

$= (b+c)^2$

Hence $a+b$, $b+c$ and $c+d$ are in GP.

$\boxed{14.}$ $9^{1/3} \cdot 9^{1/3^2} \cdot 9^{1/3^3} \ldots$

$= 9^{1/3 + 1/3^2 + 1/3^3 + \cdots} = 9^{1/3 \cdot \frac{1}{1-1/3}} = 9^{1/2} = 3$

$\boxed{15.}$ Since a, b, c and d are in GP

Then $\cancel{\frac{b}{a}}$ $b^2 = ac$, $c^2 = bd$. $\cancel{c^2 = b}$ $bc = ad$

Consider $(a^n + b^n)(c^n + d^n)$

$= (ac)^n + (ad)^n + (bc)^n + (bd)^n$

$= b^{2n} + c^{2n} + b^n c^n + b^n c^n = (b^n + c^n)^2$

Hence $a^n + b^n$, $b^n + c^n$, $c^n + d^n$ are in GP.

16.

$0.5 + 0.55 + 0.555 + \cdots$ n terms

$= 5\,[0.1 + 0.11 + 0.111 + \cdots \text{ n}^{th} \text{ term}]$

$\frac{9}{10} = \frac{10-1}{10}$

$1 - 0.1$

$1.00 - 0.01$

$= \frac{5}{9}\left[0.9 + 0.99 + 0.999 + \cdots\right]$

$= \frac{5}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \cdots + \left(1 - \frac{1}{10^n}\right)\right]$

$= \frac{5}{9}\left[n - \frac{1}{10}\left(\frac{1 - \frac{1}{10^n}}{1 - \frac{1}{10}}\right)\right] = \frac{5}{9}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$

$= \frac{5}{9}n - \frac{5}{81}\left(1 - \frac{1}{6^n}\right)$

17.

p^{th} term $= xy^{p-1} = a$

q^{th} term $= xy^{q-1} = b$

r^{th} term $= xy^{r-1} = c$

1st term $= x$

Common ratio $= y$

Then $a^{q-r}\, b^{r-p}\, c^{p-q}$

$= (xy^{p-1})^{q-r}\,(xy^{q-1})^{r-p}\,(xy^{r-1})^{p-q}$

$= x^{q-r}\, y^{(p-1)(q-r)}\, x^{r-p}\, y^{(q-1)(r-p)}\, x^{p-q}\, y^{(r-1)(p-q)}$

$= 1.$

18. Let a and b are two number.

Then ATQ $a + b = 6\sqrt{a.b}$

$\Rightarrow \cancel{(a+b)^6 = 36ab}$

$(a-b)^2 = (a+b)^2 - 4ab = 36ab - 4ab = 32ab$

$\Rightarrow a-b = \pm 4\sqrt{2}\sqrt{ab}$

$\cancel{a+b = 6\sqrt{ab}}$
$\cancel{a-b = 4\sqrt{2}\sqrt{ab}}$
$\cancel{2a =}$

now $\dfrac{a+b}{a-b} = \dfrac{6\sqrt{ab}}{\pm 4\sqrt{2}\sqrt{ab}} = \dfrac{3}{\pm 2\sqrt{2}} = \dfrac{3}{2\sqrt{2}}$ or $\dfrac{3}{-2\sqrt{2}}$

$\Rightarrow \dfrac{2a}{2b} = \dfrac{3+2\sqrt{2}}{3-2\sqrt{2}}$ or $\dfrac{3-2\sqrt{2}}{3+2\sqrt{2}}$

$\therefore a:b = 3+2\sqrt{2} : 3-2\sqrt{2}$ or $3-2\sqrt{2} : 3+2\sqrt{2}$

$\boxed{\text{OR}}$

Since in GP $S_n = a\left(\dfrac{r^n-1}{r-1}\right)$

Consider $S_{10}(S_{30}-S_{20}) = a\left(\dfrac{r^{10}-1}{r-1}\right)\left[a\left(\dfrac{r^{30}-1}{r-1}\right) - a\left(\dfrac{r^{20}-1}{r-1}\right)\right]$

$= \dfrac{a^2}{(r-1)^2}(r^{10}-1)(r^{30}-r^{20}) = a^2\dfrac{(r^{20}-r^{10})(r^{20}-r^{10})}{(r-1)^2}$

$= \left[a\dfrac{(r^{20}-1+1-r^{10})}{(r-1)}\right]^2 = \left[a\left(\dfrac{r^{20}-1}{r-1}\right) - a\left(\dfrac{r^{10}-1}{r-1}\right)\right]^2$

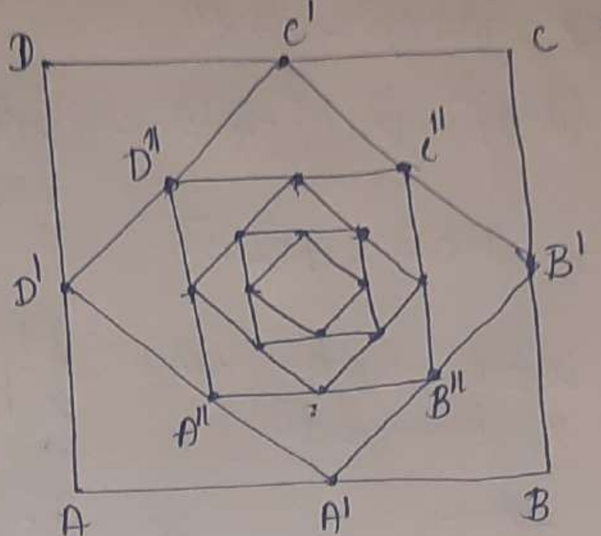
$= (S_{20}-S_{10})^2$

Hence $(S_{10}-S_{20})^2 = S_{10}(S_{30}-S_{20})$

19.

$AB = BC = CD = DA = 100$ cm

A', B', C', D' are mid-point of AB, BC, CA, DA respectively.



By SAS

$\triangle C'D'D \cong \triangle AA'D' \cong \triangle A'B'B \cong \triangle B'CC'$

Area of Quadrilateral $ABCD = 100^2 = 10{,}000$

Area of $\triangle AA'D' = \frac{1}{2} \times 50 \times 50 = \cancel{50}\,1250$

Area of $(\triangle AA'D' + \triangle A'BB' + \triangle B'CC' + \triangle C'DD') = \cancel{4 \times 50 = 200}$

$= 4 \times 1250 = 5000$

$\therefore$ Area of $A'B'C'D' = \frac{1}{2}$ area of ABCD

Similarly area of $A''B''C''D'' = \frac{1}{2}$ area of $A'B'C'D'$

$= \frac{1}{2^2}$ area of ABCD

and so on.

(a) Since $A'B'^2 = \frac{1}{2} AB^2 = \frac{1}{2} 100^2 = 5000$

$\Rightarrow A'B' = \sqrt{2 \times 5^2 \times 10^2} = 50\sqrt{2}$

(b) Again $A''B''^2 = \frac{1}{2} A'B'^2 = \frac{1}{2}(50\sqrt{2})^2$

$\Rightarrow A''B'' = \frac{1}{\sqrt{2}} 50\sqrt{2} = 50$

(iii) Perimeter of $A'B'C'D' = 4(A'B') = 4 \times \sqrt{2}\, 50$
$= 200\sqrt{2}$

(d) Area of $ABCD$ + area of $A'B'C'D'$ + ...

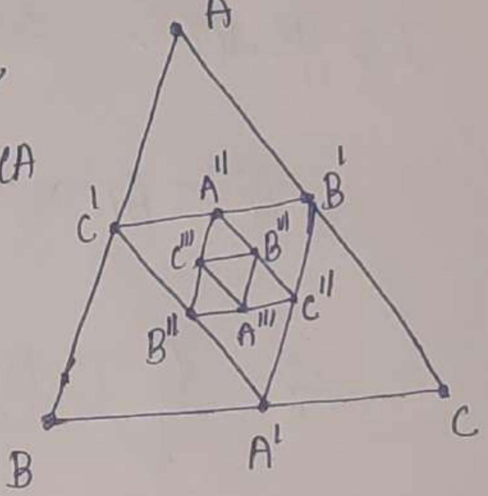
$= AB^2 + A'B'^2 + A''B''^2 + \cdots$

$= AB^2 + \frac{1}{2}AB^2 + \frac{1}{2^2}AB^2 + \cdots$

$= AB^2\left[1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots\right] = 100^2 \frac{1}{1-\frac{1}{2}}$

$= 10{,}000 \times 2 = 20{,}000$

20. Since $\triangle ABC$ is an equilateral
A', B', C' are mid-point of BC, CA and AB respectively.



By mid-point theorem

$B'C' = \frac{1}{2}BC$, $A'B' = \frac{1}{2}AB$,
$C'A' = \frac{1}{2}CA$ similaly other's.

so By S-S-S $\triangle AB'C' \cong \triangle A'BC' \cong \triangle A'B'C \cong \triangle A'B'C'$

so area of $\triangle A'B'C' = \frac{1}{4}$ area of $\triangle ABC$

similaly area of $\triangle A''B''C'' = \frac{1}{4}$ area of $\triangle A'B'C'$

$= \frac{1}{4^2}$ area of $\triangle ABC$

and so on. Area of K^{th} triangle $= \frac{1}{4^{K-1}}$ area of $\triangle ABC$

(a) $AB = BC = CA = 24\,cm$.

(i) Area of 5^{th} triangle $= \frac{1}{4^4}$ area of $\triangle ABC$

$$\frac{\sqrt{3}}{4}(\text{side})^2 = \frac{1}{256}\,\frac{\sqrt{3}}{4}AB^2 = \frac{\sqrt{3}}{2^{10}}(2^3 \times 3)^2 = \frac{9}{16}\sqrt{3}$$

(ii) $\Rightarrow$ side $= \sqrt{\frac{9}{4}} = \frac{3}{2}$.

$\therefore$ side of $5^{th}\,\triangle = \frac{3}{2}$.

(c) In general side of n^{th} triangle.

~~side~~ Area of $n^{th}\,\triangle = \frac{1}{4^{n-1}}$ area of ABC

$$\Rightarrow \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{1}{4^{n-1}}\,\frac{\sqrt{3}}{4}(AB)^2$$

$$\Rightarrow \boxed{\underset{\text{of } n^{th}\triangle}{\text{side}} = \frac{1}{4^{\frac{n-1}{2}}}AB} = \frac{1}{2^{n-1}}AB$$

$2^3 \times 3$

(b) Side of $(1^{st}\triangle + 2^{nd}\triangle + 3^{rd}\triangle + 4^{th}\triangle + 5^{th}\triangle + 6^{th}\triangle)$

$$= AB + \frac{1}{2}AB + \frac{1}{2^2}AB + \frac{1}{2^3}AB + \frac{1}{2^4}AB + \frac{1}{2^5}AB$$

$$= \frac{(1-1/2^6)}{(1-1/2)}AB = \left(2 - \frac{1}{2^5}\right)24 = 48 - \frac{3}{4}$$

$$= 48 - 0.75 = 47.25$$

Hence sum of 1st 6th triangle perimeter $= 3 \times 47.25$

(c) Area of all triangle

$$= \Delta ABC + \frac{1}{4}\Delta ABC + \frac{1}{4^2}\Delta ABC + \cdots$$

$\frac{1}{3}$

$$= \frac{1}{1-\frac{1}{4}}\Delta ABC = \frac{4}{3}\,\frac{\sqrt{3}}{4}AB^2 = \frac{\sqrt{3}}{3}(2^3\times 3)^2$$

$$= 3\times 2^6 = 192 \text{ sq. cm.}$$

(d) perimeter of all triangle

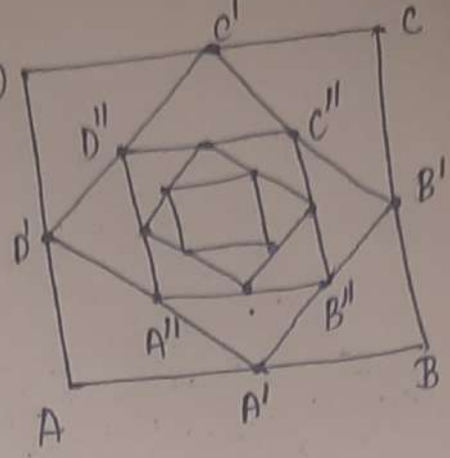
$$= 3AB + \frac{3}{2}AB + \frac{3}{2^2}AB + \frac{3}{2^3}AB + \cdots$$ 2

$$= 3AB\,\frac{1}{1-\frac{1}{2}} = 6\times 24 = 144 \text{ cm.}$$

19. Method-I (through sides)

Here $AB = BC = CD = DA = 100$ cm

A', B', C' and D' are mid-point of AB, BC, CA and DA respectively.



$A'B' = \sqrt{A'B^2 + BB'^2} = 50\sqrt{2}$ cm

where $A'B = BB' = 50$ cm

So $\frac{AB}{A'B'} = \frac{100}{50\sqrt{2}} \Rightarrow \boxed{A'B' = \frac{1}{\sqrt{2}} AB}$

similarly $A''B'' = \frac{1}{\sqrt{2}} A'B' = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} AB$

$\Rightarrow \boxed{A''B'' = \frac{1}{(\sqrt{2})^2} AB}$ and $\boxed{A'''B''' = \frac{1}{(\sqrt{2})^3} AB}$

(i) Side of $A'B'C'D'$ is $50\sqrt{2}$

(ii) area of square $A''B''C''D''$ is $A''B''^2 = \left(\frac{1}{(\sqrt{2})^2} AB\right)^2$

$= \left(\frac{1}{2} 100\right)^2 = 2500$

(iii) Perimeter of $A'B'C'D'$ is $4 \times A'B' = 4 \times 50\sqrt{2} = 200\sqrt{2}$

(iv) Sum of areas of squares

$= AB^2 + A'B'^2 + A''B''^2 + \cdots$

$= AB^2 + \frac{1}{2} AB^2 + \frac{1}{2^2} AB^2 + \frac{1}{2^3} AB^2 + \cdots$

$= AB^2 \left[1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots\right]$

$= 100^2 \frac{1}{1 - 1/2} = 2 \times 10{,}000 = 20{,}000$