JINDAL SCHOOL CLASS-XI TEST-29/10/24 $\boxed{1}$

MATHEMATICS

Section-A

1. Arithmetic mean of 4^x & 4^{1-x} is: $\frac{4^x+4^{1-x}}{2}$

Geometric mean of 4^x and 4^{1-x} is: $\sqrt{4^x \cdot 4^{1-x}}$

Since A.M $\geq$ G.M

$$\Rightarrow \frac{4^x+4^{1-x}}{2} \geq \sqrt{4^x \cdot 4^{1-x}}$$

$$\Rightarrow 4^x+4^{1-x} \geq 4 \;;\forall x \in \mathbb{R}$$

Hence minimum value of 4^x+4^{1-x} is 4.

2. Let First term $= a$

and common Difference $= d$

General term of AP is $t_n = a+(n-1)d$

$t_9 = a+(9-1)d = a+8d$

and $t_{13} = a+(13-1)d = a+12d$

Since $9t_9 = 13t_{13}$

$$\Rightarrow 9(a+8d) = 13(a+12d)$$

$$\Rightarrow 9a+72d = 13a+156d$$

$$\Rightarrow 4a+84d=0 \Rightarrow a+21d=0$$

$\Rightarrow \boxed{t_{22} = 0}$

∴ 22^{nd} term of an AP is 0.

3. Let First term $= a$

and Common ratio $= r$

Since $t_n = ar^{n-1}$

Given $t_3 = 4$

$\Rightarrow ar^2 = 4$

Again ~~product~~ $t_1 t_2 t_3 t_4 t_5$

$= a . ar . ar^2 . ar^3 . ar^4$

$= a^5 r^{10} = (ar^2)^5 = 4^5$

Hence product of first 5 term is 4^5.

4. Let First term $= a$

Common Difference $= d$

Then $S_n = \frac{n}{2}[2a + (n-1)d]$ and $t_n = a+(n-1)d$

$S_{2n} = 3S_n$

$\Rightarrow \frac{2n}{2}[2a+(2n-1)d] = 3\frac{n}{2}[2a+(n-1)d]$

$\Rightarrow 4a + (4n-2)d = 6a + (3n-3)d$

$\Rightarrow \boxed{2a = (n+1)d}$

again $\dfrac{S_{3n}}{S_n} = \dfrac{\frac{3n}{2}[2a+(3n-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \dfrac{3((n+1)d+(3n-1)d)}{(n+1)d+(n-1)d}$

$= 3 \cdot \dfrac{4n}{2n} = 6$

Hence $\dfrac{S_{3n}}{S_n} = 6$

5. x, y and z are in G.P

Then $y = xr$ and $z = xr^2$

where common ratio $= r$

Again $x, 2y$ and $3z$ are in A.P

Then $2(2y) = x + 3z$

$\Rightarrow 4xr = x + 3xr^2$

$\Rightarrow 3r^2 - 4r + 1 = 0$

$\Rightarrow (3r-1)(r-1) = 0 \Rightarrow$

$\Rightarrow r = 1$ or $r = 1/3$.

If ~~$r=1$ then~~ Hence Common ratio is $1/3$.

Section-B

6. Given x, y and z are in AP

Then $2y = x+z$

$\Rightarrow 4y^2 = x^2+z^2+2xz$

$\Rightarrow \boxed{\cancel{x^2+xz+z^2 = 2y^2+xz}}$

~~Similarly~~ $\Rightarrow 2x^2+2z^2+2xz = x^2+z^2+2y^2+2y \cdot y$

$\Rightarrow 2(x^2+xz+z^2) = x^2+\cancel{y^2}+z^2+y^2+y(x+z)$

$= (x^2+xy+y^2)+(z^2+yz+y^2)$

Hence x^2+xy+y^2, x^2+xz+z^2 and y^2+yz+z^2

are in A.P.

7. Given First term $= a = 2$

Common Ratio $= r = 4$

$\cancel{t_1 = 2 = a}$

$t_n = ar^{n-1} = 2\,4^{n-1} = 2^{2n-1}$

Since $t_n = 131072$

$\Rightarrow 2^{2n-1} = 2^{17}$ $\Rightarrow$

$\Rightarrow 2n-1 = 17 \Rightarrow \boxed{n=9}$

$\therefore$ 9th term of GP is 131072.

SECTION-C

8. Let three term of GP are

$\frac{a}{r}, a,$ and ar

Where First term $= a$

Common ~~Difference~~ ratio $= r$

Given $\frac{a}{r} \cdot a \cdot ar = -1$

$\Rightarrow a^3 = -1 \Rightarrow \boxed{a=-1}$

also $\frac{a}{r} + a + ar = 13/12$

$\Rightarrow \frac{1}{r} + 1 + r = -\frac{13}{12}$

$\Rightarrow 12r^2 \cancel{+25r} + 12 = 0$

$\cancel{\Rightarrow r = 1 \pm 1} \Rightarrow 12r^2 + 16r + 9r + 12 = 0$

$\Rightarrow (4r+3)(3r+4) = 0$

$\Rightarrow r = -3/4$ or $-4/3$

Hence if $a=-1$ and $r=-3/4$

Then terms are $\frac{4}{3}, -1$ & $3/4$

If $a = -1$ and $r = -4/3$

then terms are $\frac{3}{4}$, -1 and $\frac{4}{3}$

9. Let First term $= x$

Common Ratio $= u$

Then p^{th} term $= a \Rightarrow xu^{p-1} = a$

q^{th} term $= b \Rightarrow xu^{q-1} = b$

r^{th} term $= c \Rightarrow xu^{r-1} = c$

now $a^{q-r} b^{r-p} c^{p-q}$

$= (xu^{p-1})^{q-r} (xu^{q-1})^{r-p} (xu^{r-1})^{p-q}$

$= x^{q-r} u^{pq-pr-q+r} x^{r-p} u^{qr-qp-r+p} x^{p-q} u^{pu-p-rq+q}$

$= 1.$

Hence $a^{q-r} b^{r-p} c^{p-q} = 1.$

10. $7 + 77 + 777 + \cdots + 777\cdots7$ (n-terms)

$= \frac{7}{9}[9 + 99 + 999 + \cdots + 999\cdots9]$

$= \frac{7}{9}[(10-1) + (10^2-1) + (10^3-1) + \cdots + (10^n-1)]$

$= \frac{7}{9}[(10 + 10^2 + \cdots + 10^n) - (1 + \cdots + 1)]$

$= \frac{7}{9}\left[10\left(\frac{1-10^n}{1-10}\right) - n\right]$

$$= \frac{7}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$$

Hence $7 + 77 + 777 + \cdots$ n-terms $= \frac{7}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$